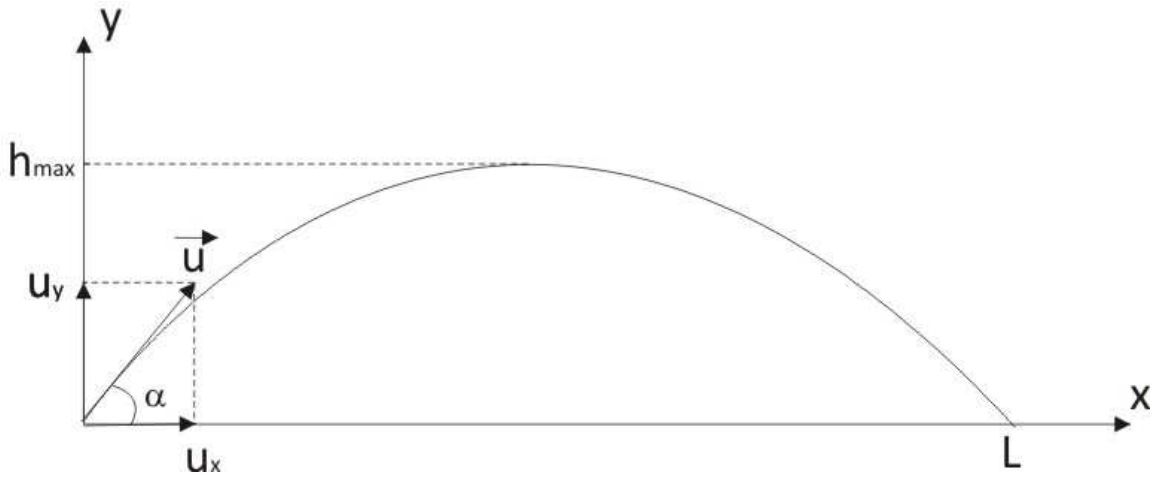
A body of mass *m* is thrown with velocity *u* at angle of 30° to the horizontal and another body B of the same mass is thrown with velocity *u* at an angle of 60° to the horizontal. Find the ratio of the horizontal range and maximum height of A and B?

Solution.

$$m, u, \alpha_A = 30°, \alpha_B = 60°;$$

$$\frac{L_A}{h_{maxA}} - ? \frac{L_B}{h_{maxB}} - ?$$

The body is thrown with velocity u at angle of α.



$$u_x = ucos\alpha;$$

$$u_y = usin\alpha.$$

The max height attained.

$$h_{max} = \frac{v_y^2 - u_y^2}{-2g}.$$

At the max height $v_y = 0$:

$$h_{max} = \frac{-u_y^2}{-2g};$$

$$h_{max} = \frac{u_y^2}{2g};$$

$$h_{max} = \frac{u^2 sin^2\alpha}{2g}.$$

The time of flight.

$$h = u_y t - \frac{gt^2}{2};$$

$$h = usin\alpha t - \frac{gt^2}{2}.$$

At the end of the flight $h = 0$:

$$0 = usin\alpha t - \frac{gt^2}{2};$$

$$usin\alpha t = \frac{gt^2}{2};$$

$$usin\alpha = \frac{gt}{2};$$

$$t = \frac{2usin\alpha}{g}.$$

The max horizontal range of the body.

$$L = u_x t;$$

$$L = ucos\alpha t;$$

$$L = \frac{2u^2 sin\alpha cos\alpha}{g}.$$

A) The body A of mass *m* is thrown with velocity *u* at angle of $\alpha_A = 30°$ to the horizontal.

The max height attained by the body A:

$$h_{maxA} = \frac{u^2 sin^2\alpha_A}{2g}.$$

The max range of the body A:

$$L_A = \frac{2u^2 sin\alpha_A cos\alpha_A}{g}.$$

The ratio of the horizontal range and maximum height of body A:

$$\frac{L_A}{h_{maxA}} = \frac{2u^2 sin\alpha_A cos\alpha_A 2g}{gu^2 sin^2\alpha_A} = \frac{4cos\alpha_A}{sin\alpha_A} = \frac{4}{tan\alpha_A} = \frac{4}{tan30°} = 6.9.$$

B) The body B of mass *m* is thrown with velocity *u* at angle of $\alpha_B = 60°$ to the horizontal.

The max height attained by the body B:

$$h_{maxB} = \frac{u^2 sin^2\alpha_B}{2g}.$$

The max range of the body B:

$$L_B = \frac{2u^2 sin\alpha_B cos\alpha_B}{g}.$$

The ratio of the horizontal range and maximum height of body B:

$$\frac{L_B}{h_{maxB}} = \frac{2u^2 sin\alpha_B cos\alpha_B 2g}{gu^2 sin^2\alpha_B} = \frac{4cos\alpha_B}{sin\alpha_B} = \frac{4}{tan\alpha_B} = \frac{4}{tan60°} = 2.3.$$

Answer:

The ratio of the horizontal range and maximum height of body A is $\frac{L_A}{h_{maxA}} = 6.9$.

The ratio of the horizontal range and maximum height of body A is $\frac{L_B}{h_{maxB}} = 2.3$.