Task. A body covers 4m in the 3rd second and 12m in the 5th second. If the motion is uniformly accelerated, how far will it travel the next 3 second.

Solution. Let v be the initial velocity of the body and a be its acceleration. Then the distance covered by the body at time t is given by the following formula:

$$d(t) = vt + \frac{gt^2}{2}.$$

We should find the distance covered by the body at next 3 seconds after 5 seconds, i.e. at time $t = 5 + 3 = 8$ s.

By assumption

$$d(3\ s) = 4\ m$$

and

$$d(5\ s) = 12\ m.$$

Thus

$$4 = d(3) = v * 3 + \frac{g * 3^2}{2} = 3v + 4.5g$$

and

$$12 = d(5) = v * 5 + \frac{g * 5^2}{2} = 5v + 12.5g$$

So we obtain the following system of equations

$$\left\{ \begin{array}{rcl} 3v + 4.5g & = & 4 \\ 5v + 12.5g & = & 12 \end{array} \right. \Rightarrow \left\{ \begin{array}{rcl} 6v + 9g & = & 8 \\ 10v + 25g & = & 24 \end{array} \right.$$

Multiplying both sides of the first equation by (-5), both sides of the second equation by 3 and adding them we obtain

$$\left\{ \begin{array}{rcl} -30v - 45g & = & -40 \\ 30v + 75g & = & 72 \end{array} \right.$$

$$-30v - 45g + 30v + 75g = 72 - 40$$

$$30g = 32$$

$$g = \frac{32}{30} = \frac{16}{15} \approx 1.0667\ m/s^2.$$

Therefore

$$3v = 4 - 4.5g$$

and so

$$v = \frac{4 - 4.5g}{3} = \frac{4 - \frac{9}{2} \cdot \frac{16}{15}}{3} = \frac{4 - \frac{3*8}{5}}{3} = \frac{4 - 4.8}{3} = -\frac{0.8}{3} = -\frac{8}{30} = -\frac{4}{15} \approx -0.26667\ m/s.$$

Hence

$$d(8) = -\frac{4}{15} * 8 + \frac{16}{15} * \frac{8^2}{2} = -\frac{32}{15} + \frac{16 * 32}{15} = \frac{16 * 32 - 32}{15} = \frac{15 * 32}{15} = 32\ m.$$

Answer. 32 m.