Task. A boy on a $h = 20\ m$ high cliff drops a stone. One second later, he throws down another stone such that both the stones hit the ground simultaneously. Find the initial velocity of the second stone. ($g = 10\ m/s^2$)

Solution. The first stone moved with initial zero velocity and acceleration $g = 10\ m/s^2$. Therefore its height at time t is given by the formula

$$h_1(t) = h - \frac{gt^2}{2}.$$

The time T when he reach the ground satisfies the following equation:

$$h_1(T) = 0 = h - \frac{gT^2}{2},$$

whence

$$\frac{gT^2}{2} = h$$

$$T = \sqrt{\frac{2h}{g}}.$$

Substituting the values we get

$$T = \sqrt{\frac{2 * 20}{10}} = \sqrt{4} = 2\ s.$$

The second stone moved with some initial velocity v and the same acceleration $g = 10\ m/s^2$. Since it starts one second later, its height at time t is given by the formula

$$h_2(t) = h - v(t-1) - \frac{g(t-1)^2}{2}.$$

Since both stones reach the ground simultaneously, we have also that

$$h_2(t) = 0 = h - v(T-1) - \frac{g(T-1)^2}{2},$$

whence

$$v(T-1) = h - \frac{g(T-1)^2}{2}$$

$$v = \frac{h - \frac{g(T-1)^2}{2}}{T-1}$$

$$v = \frac{h}{T-1} - \frac{g(T-1)}{2}$$

Substituting the values of h, g, and T we get

$$v = \frac{h}{T-1} - \frac{g(T-1)}{2} = \frac{20}{2-1} - \frac{10 * (2-1)}{2} = 20 - 5 = 15\ m/s.$$

Answer. $v = 15\ m/s$.