Task. The position vector of a particle is $r(t) = (t^2 - 1)i + 2tj$. How can I find out the trajectory of the particle in the XY-plane ?

Solution. Let $x(t) = t^2 - 1$ and $y(t) = 2t$ be the coordinate functions of the vector, so

$$r(t) = (t^2 - 1)i + 2tj = x(y)i + y(t)j.$$

Notice that we can express t via $y(t)$:

$$y(t) = 2t,$$

whence

$$t = y/2.$$

Substituting this formula into the equation of $x(t)$ we obtain

$$x = t^2 - 1$$
$$x = (y/2)^2 - 1$$
$$x = \frac{y^2}{4} - 1$$
$$4x = y^2 - 4$$
$$y^2 - 4x = 4.$$

The trajectory of the particle is given by the following equation

$$y^2 - 4x = 4.$$