Task. A railroad car of mass M moving at a speed v1 collides and couples with two coupled railroad cars, each of the same mass M and moving in the same direction at a speed v2.

(b) How much kinetic energy is lost in the collision? Answer in terms of M, v1, and v2.

Solution. We will use Momentum Conservation Low.

The momentum of the first railroad car before collision is equal to

$$p_1 = Mv_1,$$

and the momentum of the pair of cars is

$$p_2 = (M + M)v_2 = 2Mv_2.$$

After collision, since the first car is couples with two others, the tree cars will move with the same velocty, say, v. Then thier impulse will be equal to

$$p = 3Mv.$$

By the Momentum Conservation Low

$$p_1 + p_2 = p,$$

so

$$Mv_1 + 2Mv_2 = 3Mv,$$
$$3v = v_1 + 2v_2,$$
$$v = \frac{1}{3}(v_1 + 2v_2).$$

The kinetic energy of the first car before collision will be

$$W_1 = \frac{Mv_1^2}{2},$$

and the kinetic energy of the pair of cars is

$$W_2 = \frac{2Mv_2^2}{2}.$$

After collision the total kinetic energy of all 3 cars is equal to

$$W = \frac{3Mv^2}{2}.$$

Therefore the loss of kinetic energy is equal to

$$\Delta W = W - (W_1 + W_2)$$

Substituting values we obtain

$$\begin{aligned}
\Delta W = W - (W_1 + W_2) &= \frac{3Mv^2}{2} - \left(\frac{Mv_1^2}{2} + \frac{2Mv_2^2}{2}\right) \\
&= \frac{3M \cdot \left(\frac{v_1+2v_2}{3}\right)^2}{2} - \frac{Mv_1^2}{2} - \frac{2Mv_2^2}{2} \\
&= \frac{M(v_1 + 2v_2)^2}{6} - \frac{Mv_1^2}{2} - \frac{2Mv_2^2}{2} \\
&= \frac{M(v_1^2 + 4v_1v_2 + 4v_2^2)}{6} - \frac{3Mv_1^2}{6} - \frac{6Mv_2^2}{6} \\
&= \frac{M}{6}\left(v_1^2 + 4v_1v_2 + 4v_2^2 - 3v_1^2 - 6v_2^2\right) \\
&= \frac{M}{6}\left(4v_1v_2 + 2v_2^2 - 2v_1^2\right) \\
&= \frac{M}{3}\left(2v_1v_2 + v_2^2 - v_1^2\right).
\end{aligned}$$

Answer. $\Delta W = \frac{M}{3}\left(2v_1v_2 + v_2^2 - v_1^2\right)$.