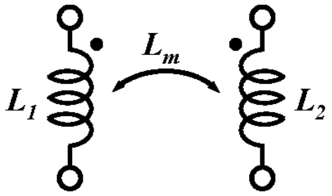
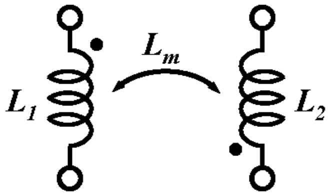
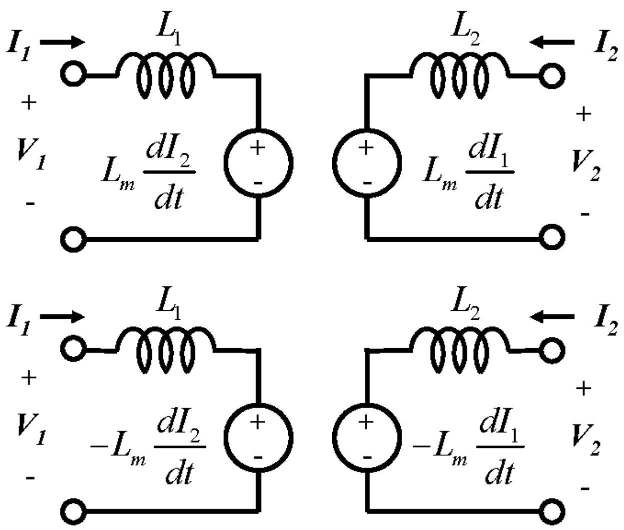
Mutual Inductance





$$V_1 = L_1 \frac{dI_1}{dt} \pm L_m \frac{dI_2}{dt}$$

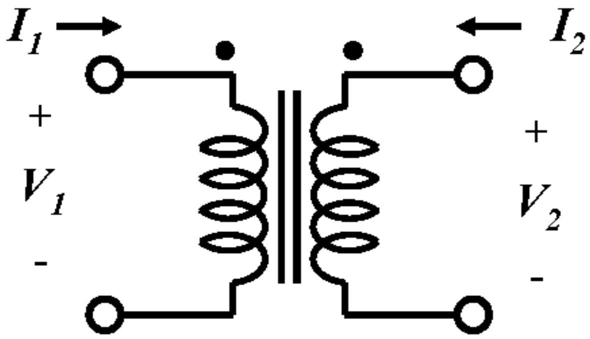
$$V_2 = \pm L_m \frac{dI_1}{dt} + L_m \frac{dI_2}{dt}$$



- Correct sign for mutual inductance found from Lenz' law and dot convention
- Dot convention: current flowing into one dot will induce current flow out of second dot

Transformers

- A transformer is just a special case where the mutual inductance is made as large as possible by allowing both coils to share the same flux
- This is usually achieved by winding them both on a common core of high permeability material (soft iron or ferrite materials)



$$V_1 = j\omega L_1 I_1 + j\omega L_m I_2$$
$$V_2 = j\omega L_m I_1 + j\omega L_2 I_2$$

When there is no flux leakage, the mutual inductance is related to the primary and secondary inductances as

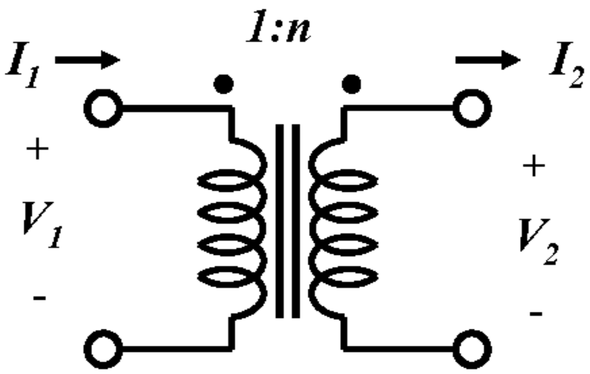
$$L_m = \sqrt{L_1 L_2}$$

For real transformers this can never be quite achieved, so we write

$$L_m = k\sqrt{L_1 L_2}$$

where $0 < k < 1$ - coefficient of coupling

Ideal Transformer



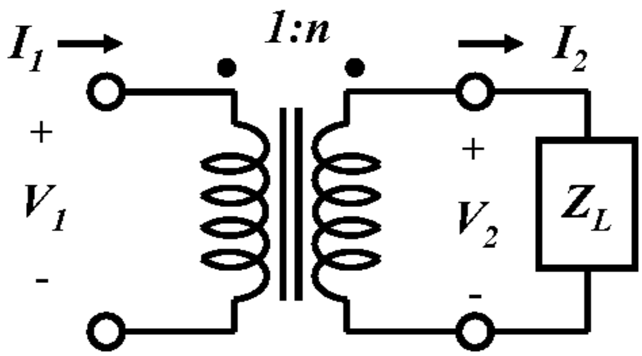
If both coils share the same flux, then Farady's law gives:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{1}{n}$$

As the permeability of the core increases, the relationship between the primary and secondary currents approaches a limiting value set by the turns ratio:

$$\frac{I_1}{I_2} \Rightarrow \frac{N_2}{N_1} = n$$

These two relationships define an ideal transformer. This is a fictitous element (note that $\mu \to \infty$ implies infinite inductances so the impedance matrix is infinite) but a real transformer approximates this behavior.



An idea transformer has the following useful property when one winding is terminated:

$$Z_m = \frac{V_1}{I_1} = \frac{{}^{N_1}\!/_{N_2}\, V_2}{{}^{N_2}\!/_{N_1}} = \frac{Z_L}{n^2}$$