

Suppose the motion of the particle is described by the equation: $x = a + bt^2$, where $a = 20\text{cm}$ and $b = 4\text{cm}\cdot\text{s}^{-2}$. a) Find the displacement of the particle in the time interval between $t_1 = 2\text{s}$ and $t_2 = 5\text{s}$. b) Find the average velocity in this time interval. c) Find the instantaneous velocity at time $t_1 = 2\text{s}$.

Solution: the equation of motion for a particle and the equation of motion in the general form:

$$x = a + bt^2, \quad \text{general: } x = x_0 + V_0t + \frac{At^2}{2}, \text{ where } A - \text{acceleration}, V_0 - \text{initial velocity}$$

Comparing the two equations we can find the initial speed, position and the acceleration:

$$x_0 = a, \quad V_0 = 0, \quad A = 2b$$

A: Because the coordinate of the particle is increasing, we will find the position of a particle at point 1 and point 2 and subtract the values of the coordinates. The difference will be equal to the displacement of the particle.

$$x_{t_1} = a + bt_1^2, \quad x_{t_2} = a + bt_2^2$$

$$\Delta S = x_{t_2} - x_{t_1} = a + bt_2^2 - a - bt_1^2 = b(t_2^2 - t_1^2) = 4 \frac{\text{cm}}{\text{s}^2} * (5\text{s} * 5\text{s} - 2\text{s} * 2\text{s}) = 84 \text{ cm}$$

B: Average speed is equal to the length of way divided by the time in which the particle passes this way:

$$V_a = \frac{\Delta S}{\Delta t}$$

$$\Delta t = t_2 - t_1$$

$$V_a = \frac{\Delta S}{\Delta t} = \frac{84 \text{ cm}}{5\text{s} - 2\text{s}} = 28 \frac{\text{cm}}{\text{s}}$$

C: The equation of motion for a particle:

$$V = V_0 + At, \quad V_0 = 0, \quad A = 2b = 8 \frac{\text{cm}}{\text{s}^2}$$

Instantaneous velocity at time $t = 2\text{s}$:

$$V_i = At = 8 \frac{\text{cm}}{\text{s}^2} * 2\text{s} = 16 \frac{\text{cm}}{\text{s}}$$

Answer: a) 84 cm b) $28 \frac{\text{cm}}{\text{s}}$, c) $16 \frac{\text{cm}}{\text{s}}$.

