A light flexible rope is wrapped several times around a solid cylinder of mass 50 kg and diameter 0.12m, which rotates without friction about fixed horizontal axis. The free the end of the rope is pulled with a constant force of magnitude 9N for a distance of 2m. If the cylinder was initially at rest, find its final angular velocity and the final speed of the rope.

Newton's Laws for Rotation:

$$I\beta = F * \left(\frac{d}{2}\right)$$

I – moment of inertia, $I = m\left(\frac{d}{2}\right)^2 = \frac{md^2}{4}$

β – angular acceleration

$$\beta = F * \frac{\left(\frac{d}{2}\right)}{m\left(\frac{d}{2}\right)^2} = \frac{2F}{md}$$

Angular velocity equals:

$\omega = \beta t \quad => \quad t = \frac{\omega}{\beta}$

And angle equals:

$$\varphi = \frac{\beta t^2}{2} = \frac{\omega^2}{2\beta}$$

Distance equals:

$l = \varphi * \left(\frac{d}{2}\right) \quad => \quad \varphi = \frac{2l}{d}$

Therefore:

$2\frac{l}{d} = \frac{\omega^2}{2\beta} \quad => \quad \omega = 2\sqrt{\frac{\beta l}{d}} = 2\sqrt{\frac{2F}{md}\frac{l}{d}} = 2\sqrt{\frac{2Fl}{md^2}}$

$$\omega = 2\sqrt{\frac{2Fl}{md^2}} = 14.14\frac{rad}{sec}$$

Final speed of rope equals: $v = \omega * \left(\frac{d}{2}\right) = \sqrt{\frac{2Fl}{m}} = \frac{3\sqrt{2}}{5} = 0.85\frac{m}{s}$

Answer: $\omega = 14.14 \frac{rad}{sec}$, $v = 0.85 \frac{m}{s}$