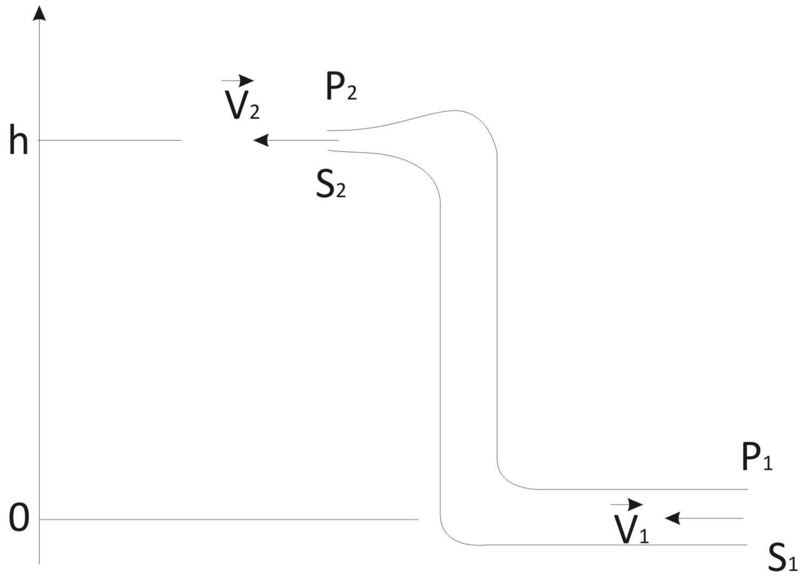
A water at a gauge pressure of 3.8 atm at street flows into an office building at a speed of 0.60 m/s through a pipe 50 mm in diameter. The pipe tapers down to 2.6 cm in diameter by the top floor, 18m above the street, where a faucet has been left open. Calculate the flow velocity and the gauge pressure in such a pipe at the top floor.

Solution.

$$P_1 = 3.8\ atm, v_1 = 0.60\frac{m}{s}, d_1 = 50mm = 50 \cdot 10^{-3}m, d_2 = 2.6cm = 2.6 \cdot 10^{-2}m,$$

$$h = 18m, g = 9.8\frac{m}{s^2}, \rho = 1000\frac{kg}{m^3};$$

$$v_2 - ?\ P_2 - ?$$



For the water in the pipe:

$$v_1 S_1 = v_2 S_2;$$

S_1 – the sectional area of the pipe at the street;
S_2 – the sectional area of the pipe by the top floor;
v_1 - the flow velocity at the street;
v_2 - the flow velocity by the top floor.

$$S_1 = \pi\frac{d_1^2}{4};$$

$$S_2 = \pi\frac{d_2^2}{4}.$$

$$v_1 \pi\frac{d_1^2}{4} = v_2 \pi\frac{d_2^2}{4};$$

$$v_1 d_1^2 = v_2 d_2^2;$$

The flow velocity by the top floor:

$$v_2 = v_1 \frac{d_1^2}{d_2^2}.$$

$$v_2 = 0.60\frac{m}{s} \cdot \frac{(50 \cdot 10^{-3} m)^2}{(2.6 \cdot 10^{-2} m)^2} = 2.22\frac{m}{s}.$$

By Bernoulli's principle:

$$\frac{\rho v_1^2}{2} + \rho g h_1 + P_1 = \frac{\rho v_2^2}{2} + \rho g h_2 + P_2;$$

ρ - the density of the water;
g - the acceleration due to gravity;
h_1 – the elevation of the pipe at the street
h_2 – the elevation of the pipe by the top floor
P_1 – the gauge pressure at the street;
P_2 – the gauge pressure by the top floor.

$$h_1 = 0;$$

$$h_2 = h.$$

$$\frac{\rho v_1^2}{2} + P_1 = \frac{\rho v_2^2}{2} + \rho g h + P_2;$$

$$P_2 = P_1 + \frac{\rho}{2}(v_1^2 - v_2^2) - \rho g h.$$

Converting the gauge pressure to *Pa*:

$$P_2 = 3.8\, atm \cdot \left(1.01 \cdot 10^5 \frac{Pa}{atm}\right) = 3.838 \cdot 10^5 Pa.$$

The gauge pressure by the top floor:

$$P_2 = 3.838 \cdot 10^5 Pa + \frac{1000\frac{kg}{m^3}}{2}\left(\left(0.60\frac{m}{s}\right)^2 - \left(2.22\frac{m}{s}\right)^2\right) - 1000\frac{kg}{m^3} \cdot 9.8\frac{m}{s^2} \cdot 18m = \\ = 205115.8Pa.$$

Converting the gauge pressure to *atm*:

$$P_2 = \frac{205115.8Pa}{\left(1.01 \cdot 10^5 \frac{Pa}{atm}\right)} = 2.03atm.$$

Answer:

The flow velocity in such a pipe at the top floor is $v_2 = 2.22\frac{m}{s}$.
The gauge pressure in such a pipe at the top floor is $P_2 = 2.03atm$.