

Task. Six capacitors each of 1 micro farad capacitance are there. How can they be grouped to have an effective capacitance 0.75 micro farad?

Solution. Recall that if n capacitors with capacitances C_1, C_2, \dots, C_n are connected in parallel, then the effective capacitance is equal to

$$C_{par} = C_1 + C_2 + \dots + C_n.$$

On the other hand, if they are connected in series, then the effective capacitance satisfies the following identity:

$$\frac{1}{C_{ser}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}.$$

In particular, if $C_1 = C_2 = \dots = C_n = C$, then

$$C_{par} = C_1 + C_2 + \dots + C_n = nC,$$

while for C_{ser} we have that

$$\frac{1}{C_{ser}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} = \frac{n}{C},$$

whence

$$C_{ser} = \frac{C}{n}.$$

These observations allow to construct the capacitance 0.75 micro farad from 6 capacitors each of 1 micro farad in the following way.

Notice that

$$0.75 = \frac{3}{4} = \frac{1}{4} + \frac{1}{2}.$$

The capacitance $\frac{1}{4}$ micro farad can be obtained by connecting 4 capacitors in series, while for the capacitance $\frac{1}{2}$ micro farad we should connect in series 2 capacitors. To get the sum of capacitances $\frac{1}{4} + \frac{1}{2}$ we should connect these two groups parallel. Thus the resulting grouping will have the following form:

