Task.

1. A train travels $s_1 = 20$ km at a uniform speed of $v = 60$ kmph and the next $s_2 = 20$ km at a uniform speed of $v_2 = 80$ kmph. Calculate the average speed.

2. A bus travels $s_1 = 40$ km at a speed of $v_1 = 50$ kmph and next $s_2 = 4$ m at a speed of $v_2 = 30$ kmph. Calculate its average speed.

Solution. 1) Let t_1 and t_2 be the times when train moved by the first and by the second parts of the path.

Thus

$$t_1 = \frac{s_1}{v_1}, \qquad t_2 = \frac{s_2}{v_2}.$$

Then by definition the average speed is equal to

$$v_{av} = \frac{s_1 + s_2}{t_1 + t_2}.$$

Substituting expressions for t_1 and t_2 into the latter formula we will get the formula for average speed:

$$v_{av} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} = \frac{(s_1 + s_2) v_1 v_2}{s_1 v_2 + v_1 s_2}.$$

Now substitute the values s_1, s_2, v_1 and v_2:

$$v_{av} = \frac{(20 + 20) * 60 * 80}{20 * 80 + 60 * 20} = \frac{192000}{2800} = 68.57 \ kmph.$$

2) Problem 2 is similar to 1) and differs only by values of distances and velocities. In particular, we will get the same fomula for the average velocity:

$$v_{av} = \frac{(s_1 + s_2) v_1 v_2}{s_1 v_2 + v_1 s_2}.$$

Substituting values we get:

$$v_{av} = \frac{(40 + 4) * 50 * 30}{40 * 30 + 50 * 40} = \frac{66000}{3200} = 20.63 \ kmph.$$