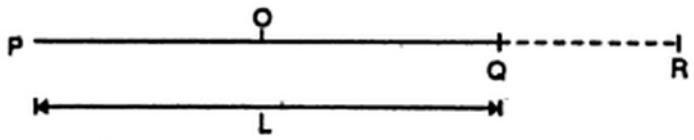
Solve.

Two ends of a train moving with a constant acceleration pass a certain point with velocities u and v. What is the velocity with which the middle point of the train passes the same point?

Solution.



Let P and Q be the two ends of the train and O is the midpoint of PQ. Let PQ = L the length of the train.

When the end Q crosses a certain point say R, the velocity of every point of the train = velocity of $Q = u$.

Hence, at the instant, the velocity of O = velocity of $P = u$.

When the end P comes to R its velocity $= v$.

For the motion of P:

$v^2 = u^2 + 2aL$ (1)

where $a = const.acceleration$

When the point O comes to R, its velocity w is given by

$w^2 = u^2 + 2a\left(\frac{L}{2}\right) = u^2 + aL$ (2)

From (1) and (2):

$$aL = \frac{v^2 - u^2}{2}$$

$$w^2 = u^2 + \left(\frac{v^2 - u^2}{2}\right) = \frac{v^2 + u^2}{2}$$

Or

$$w = \sqrt{\frac{v^2 + u^2}{2}}$$

Answer:

$$w = \sqrt{\frac{v^2 + u^2}{2}}$$