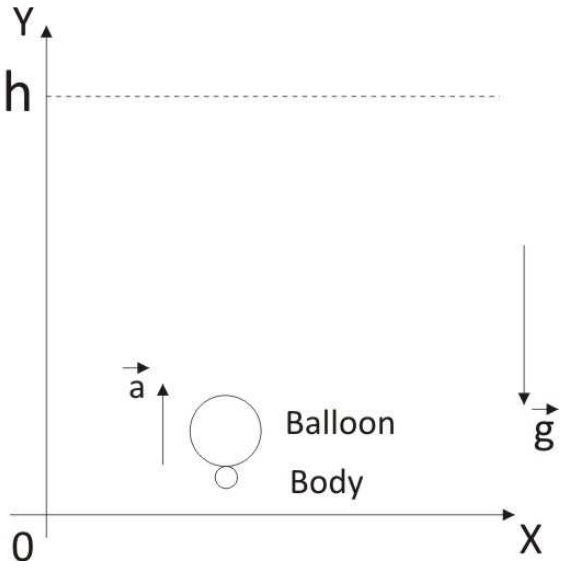
A balloon starts rising from ground with an acceleration of 1.25m/s^2. After 8sec a body is released from it. What is the time taken by the body to reach the ground?

Solution.

$$a = 1.25\frac{m}{s^2}, t_1 = 8s, g = 9.8\frac{m}{s^2};$$

$$t_2 - ?$$



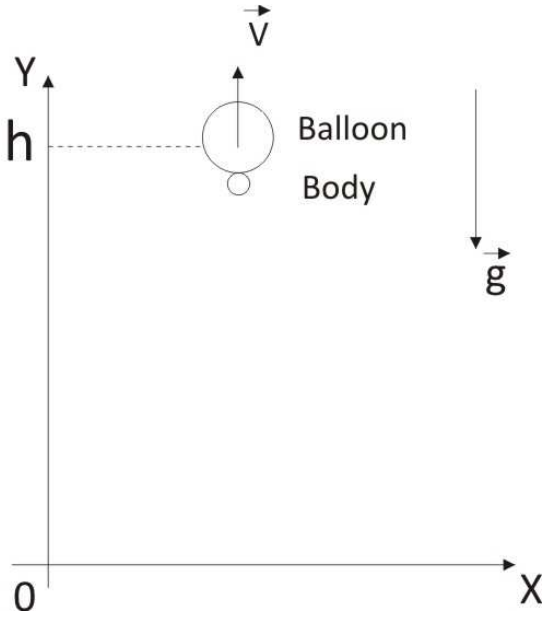
An equation of motion of a balloon:

$$h = \frac{at_1^2}{2}.$$

The speed of the balloon at the height h:

$$v = at_1.$$

A body has the same speed $v = at_1$ as a balloon.



An equation of motion of a body:

$$0 = h + vt_2 - \frac{gt_2^2}{2};$$

$$0 = \frac{at_1^2}{2} + at_1t_2 - \frac{gt_2^2}{2};$$

$$0 = \frac{at_1^2}{g} + \frac{2at_1}{g}t_2 - t_2^2.$$

t_2 - the time taken by the body to reach the ground.

There is a quadratic equation, where t_2 represents an unknown:

$$t_2^2 - \frac{2at_1}{g}t_2 - \frac{at_1^2}{g} = 0;$$

A discriminant is:

$$\Delta = \left(\frac{2at_1}{g}\right)^2 - 4 \cdot \left(-\frac{at_1^2}{g}\right) = \frac{4a^2t_1^2}{g^2} + \frac{4at_1^2}{g} = \frac{4at_1^2}{g}\left(\frac{a}{g} + 1\right).$$

$$t_2' = \frac{\frac{2at_1}{g} + \sqrt{\Delta}}{2} = \frac{\frac{2at_1}{g} + \sqrt{\frac{4at_1^2}{g}\left(\frac{a}{g} + 1\right)}}{2} = \frac{\frac{2at_1}{g} + 2t_1\sqrt{\frac{a}{g}\left(\frac{a}{g} + 1\right)}}{2} = \frac{at_1}{g} + t_1\sqrt{\frac{a}{g}\left(\frac{a}{g} + 1\right)}.$$

$$t_2' = \frac{1.25 \cdot 8}{9.8} + 8\sqrt{\frac{1.25}{9.8}\left(\frac{1.25}{9.8} + 1\right)} = 4.05(s).$$

$$t_2'' = \frac{\frac{2at_1}{g} - \sqrt{\Delta}}{2} = \frac{\frac{2at_1}{g} - \sqrt{\frac{4at_1^2}{g}\left(\frac{a}{g} + 1\right)}}{2} = \frac{\frac{2at_1}{g} - 2t_1\sqrt{\frac{a}{g}\left(\frac{a}{g} + 1\right)}}{2} = \frac{at_1}{g} - t_1\sqrt{\frac{a}{g}\left(\frac{a}{g} + 1\right)}.$$

$$t_2' = \frac{1.25 \cdot 8}{9.8} - 8\sqrt{\frac{1.25}{9.8}\left(\frac{1.25}{9.8} + 1\right)} = -2.01(s).$$

The time cannot be negative, then $t_2 = t_2' = 4.05s$.

Answer: The time taken by the body to reach the ground is $t_2 = 4.05s$.