

A sphere of radius 'R' is rolling with angular velocity 'w'. Find 'w' so that it can just climb the step of height 'h'.

The law of conservation of energy:

$$T + U = \text{const}$$

T - kinetic energy

$$T = \frac{mv^2}{2} + \frac{Iw^2}{2}$$

where:

m - mass of the body

v - speed of center mass

I - moment of inertia (for sphere equals $\frac{2}{3}mR^2$)

w - angular velocity

If it rolling = wR , therefore

$$T = \frac{1}{2}mw^2R^2 + \frac{1}{3}mw^2R^2 = \frac{5}{6}mw^2R^2$$

$U = mgh$ - potential energy

where:

g - gravitational acceleration

h - high

$$T_1 + U_1 = T_2 + U_2$$

1 - initial state

2 - final state

$$U_1 = 0, T_2 = 0:$$

$$T_1 = U_2$$

Therefore:

$$\frac{5}{6}mw^2R^2 = mgh$$

Finally:

$$w = \sqrt{\frac{6gh}{5R^2}}$$

$$\text{Answer: } w = \sqrt{\frac{6gh}{5R^2}}$$