

An accelerated body covers distance a, b, c in time l, m, n , respectively.

Show that $a \cdot (m-n) + b \cdot (n-l) + c \cdot (l-m) = 0$.

Solution: Distance s , traveled by accelerated body is equal to $s = v_0 \cdot t + \frac{a \cdot t^2}{2}$, where a is the acceleration of the body, v_0 is the initial velocity of the body and t is the time of movement. We assume that initial velocity is zero. Then, $a = \frac{a \cdot l^2}{2}$. After time l velocity of body will be equal to $v_l = a \cdot l$, and then $b = v_l \cdot m + \frac{a \cdot m^2}{2} = a \cdot l \cdot m + \frac{a \cdot m^2}{2} = a \cdot m \cdot (l + m/2)$. After time m velocity of body will be equal to $v_m = v_l + a \cdot m = a \cdot l + a \cdot m = a \cdot (l + m)$, and then $c = v_m \cdot n + \frac{a \cdot n^2}{2} = a \cdot (l + m) \cdot n + \frac{a \cdot n^2}{2} = a \cdot n \cdot (l + m + n/2)$.

We can substitute these formulas into initial equation:

$$\begin{aligned} & \frac{a \cdot l^2}{2} \cdot (m-n) + a \cdot m \cdot (l+m/2) \cdot (n-l) + a \cdot n \cdot (l+m+n/2) \cdot (l-m) = \\ & = a \cdot \left[\frac{ml^2}{2} - \frac{nl^2}{2} + mnl - ml^2 + \frac{nm^2}{2} - \frac{lm^2}{2} + nl^2 - mnl + mnl - nm^2 + \frac{ln^2}{2} - \frac{mn^2}{2} \right] = \\ & = a \cdot \left[\frac{nl^2}{2} + mnl - \frac{ml^2}{2} - \frac{lm^2}{2} - \frac{nm^2}{2} + \frac{ln^2}{2} - \frac{mn^2}{2} \right] = a \cdot \left[mnl + \frac{n(l^2 - m^2) - m(l^2 + n^2) + l(n^2 - m^2)}{2} \right] \neq 0 \end{aligned}$$

As you see, the result of such formula where $0 < l < m < n$ can't be equal to zero. We can make a conclusion that initial formula is false, or that condition of the question isn't full.