An electron accelerated through 30 kV in a cathode ray oscilloscope enters a system of deflecting plates. The deflecting field between the plates is 24 kV/m, the length of the deflecting plate is 0.06 m and the total deflection produced in the path of electron on the screen is 9 mm. What is the distance of the screen from near end of the plate?

Solution: Horizontal speed of electron v_h before deflecting can be calculated from the law of conservation of energy: $e \cdot V = \frac{m_e \cdot v_h^2}{2}$, $v_h = \sqrt{\frac{2e \cdot V}{m_e}} = \sqrt{\frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 30 \cdot 10^3}{9.11 \cdot 10^{-31}}} = 1.03 \cdot 10^8 \frac{\text{m}}{\text{s}}$, where e is the charge of electron, V is the accelerating potential difference, m_e is the mass of electron. As you see, according to the laws of classical physics speed of the electron is bigger than 20% of the velocity of light. We must calculate his speed using the formula of relativity theory: $K = m_e \cdot c^2 \cdot (\gamma - 1)$, $\gamma = \left(1 - v_h^2/c^2\right)^{-0.5}$, where K is the kinetic energy of electron and c is the velocity of light.

Then, $K = e \cdot V = 1.6 \cdot 10^{-19} \cdot 30 \cdot 10^3 = 4.8 \cdot 10^{-15}$ J; $\gamma = \frac{K}{m_e \cdot c^2} + 1 = \frac{4.8 \cdot 10^{-15}}{9.11 \cdot 10^{-31} \cdot (3 \cdot 10^8)^2} + 1 = 1.0585$;

$v_h = c \cdot \sqrt{1 - 1/\gamma^2} = 3 \cdot 10^8 \cdot \sqrt{1 - 1/1.0585^2} = 9.84 \cdot 10^7 \frac{\text{m}}{\text{s}}$.

Mass of moving electron can be calculated as $m_e' = \frac{K}{c^2} + m_e = \frac{4.8 \cdot 10^{-15}}{(3 \cdot 10^8)^2} + 9.11 \cdot 10^{-31} = 9.64 \cdot 10^{-31}$ kg.

Time of deflecting t_d (time of flying between the plates) of the electron is $t_d = \frac{l}{v_h} = \frac{0.06}{9.84 \cdot 10^7} = 6.1 \cdot 10^{-10}$ s where l is the length of the plates.

Vertical acceleration of the moving electron is $a = \frac{F}{m_e'} = \frac{e \cdot E}{m_e'} = \frac{1.6 \cdot 10^{-19} \cdot 24 \cdot 10^3}{9.64 \cdot 10^{-31}} = 4.0 \cdot 10^{15} \frac{\text{m}}{\text{s}^2}$, where F is the applied to electron force of deflecting electric field E.

Then, vertical speed of electron after deflection will be $v_v = a \cdot t_d = 4.0 \cdot 10^{15} \cdot 6.1 \cdot 10^{-10} = 2.44 \cdot 10^6 \frac{\text{m}}{\text{s}}$.

Vertical deflection of electron in the electric field can be calculated as a distance traveled by the uniformly accelerated object: $s_e = \frac{a \cdot t_d^2}{2} = 4.0 \cdot 10^{15} \cdot (6.1 \cdot 10^{-10})^2/2 = 7.44 \cdot 10^{-4}\ \text{m} = 0.744$ mm.

The rest of the vertical distance electron travels with constant speed v_v, and then time t_u of the electron uniform motion, which is equal to the time of movement from the near end of the plate to the screen, is:

$t_u = \frac{d - s_e}{v_v} = \frac{9 \cdot 10^{-3} - 7.44 \cdot 10^{-4}}{2.44 \cdot 10^6} = 3.38 \cdot 10^{-9}$ s, where d is the total deflection of the electron.

Electron travels the distance from the near end of the plate to the screen L with constant horizontal velocity. Then, $L = t_u \cdot v_h = 3.38 \cdot 10^{-9} \cdot 9.84 \cdot 10^7 = 0.333\ \text{m} = 33.3$ cm.

Answer: 33.3 cm.