Two vessels of different material are identical in size and in dimension. They are filled with equal quantity of ice at 0C. If ice in both vessels melts completely in 15 minutes and 10 minutes, compare the thermal conductivity of the metals of both vessels?

Solution.

$$\Delta t_1 = 15min, \Delta t_2 = 10min;$$

$$\frac{k_2}{k_1} - ?$$

The integral form of Fourier's law:

$$\frac{\Delta Q}{\Delta t} = kS\frac{\Delta T}{\Delta x};$$

ΔQ – the heat that is required to melt of the ice;
Δt - the time of melting of the ice;
k - the metal's thermal conductivity;
S - the surface area of the vessel;
ΔT - the temperature difference between environment and vessel;
Δx - the wall thickness of the vessel.

The integral form of Fourier's law for the first vessel:

$$\frac{\Delta Q_1}{\Delta t_1} = k_1 S_1 \frac{\Delta T_1}{\Delta x_1}.$$

The integral form of Fourier's law for the second vessel:

$$\frac{\Delta Q_2}{\Delta t_2} = k_2 S_2 \frac{\Delta T_2}{\Delta x_2}.$$

The heat required for melting the ice in the first vessel is the same as in the second vessel then:

$$\Delta Q_1 = \Delta Q_2 = \Delta Q.$$

The temperature difference between environment and the first vessel is the same as the temperature difference between environment and the second vessel then:

$$\Delta T_1 = \Delta T_2 = \Delta T.$$

Two vessels are identical in size.

The surface area of the first vessel is the same as the surface area of the second vessel then:

$$S_1 = S_2 = S.$$

The wall thickness of the first vessel is the same as the wall thickness of the second vessel then:

$$\Delta x_1 = \Delta x_2 = \Delta x.$$

There are two equations:

First equation:

$$\frac{\Delta Q}{\Delta t_1} = k_1 S \frac{\Delta T}{\Delta x}.$$

Second equation:

$$\frac{\Delta Q}{\Delta t_2} = k_2 S \frac{\Delta T}{\Delta x}.$$

After dividing the second equation by the first, we get:

$$\frac{\Delta t_1}{\Delta t_2} = \frac{k_2}{k_1}.$$

The ratio of the thermal conductivity of the metals of both vessels is:

$$\frac{k_2}{k_1} = \frac{\Delta t_1}{\Delta t_2}.$$

$$\frac{k_2}{k_1} = \frac{15min}{10min} = 1.5.$$

Answer: The ratio of the thermal conductivity of the metals of both vessels is $\frac{k_2}{k_1} = 1.5$.