

During a test a rocket travels upward at  $75 \text{ meters/sec}^2$ , and when it is 40 meters from the ground its engine fails. Determine the maximum height  $h_0$  reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of  $9.81 \text{ meters/sec}^2$  due to gravity. Neglect the effect air resistance?

rocket travels upward with uniform acceleration:

$$h_0 = \frac{at_0^2}{2}$$

$$h_0 = 40 \text{ metres}$$

$$a = 75 \frac{\text{meters}}{\text{s}^2}$$

$t_0$  – time before engine fails

$$t_0 = \sqrt{\frac{2h_0}{a}}$$

Therefore, velocity when engine fails

$$v_0 = a * t_0 = \sqrt{2h_0 a}$$

Velocity of rocket after that equals:

$$v = v_0 - gt$$

maximum height if  $v = 0$ . Therefore:

$$t = \frac{v_0}{g}$$

$$h_{max} = h_0 + v_0 t - \frac{gt^2}{2} = h_0 + \frac{v_0^2}{2g} = h_0 + \frac{h_0 a}{g} = h_0 \left(1 + \frac{a}{g}\right) = 40 * \left(1 + \frac{75}{9.81}\right) = 345.8 \text{ m}$$

The law conservation of energy:

$$mgh_{max} = \frac{mv_m^2}{2}$$

$v_m$  – speed of rocket just before it hits the ground

$$v_m = \sqrt{2gh_0 \left(1 + \frac{a}{g}\right)} = \sqrt{2h_0(a + g)} = \sqrt{2 * 40 * (75 + 9.81)} = 82.4 \text{ m/s}$$

Answer:  $h_{max} = 345.8 \text{ m}$ ,  $v_m = 82.4 \text{ m/s}$