Question: Hi, I have been reading the brief history of time by Stephen Hawking and have been greatly enjoying it. In the second chapter he mentions the fact that time is slower when in proximity with a large mass such as the earth. So a twin on top of a mountain would age faster than his counterpart at sea level. He credits this phenomenon to the energy of light, saying that the energy of light decreases as gravity decreases. I was wondering how the energy of light can vary but its speed remain constant. Since high school I have been taught that if you change an objects energy you are also changing its gravitational potential or velocity. Any reply will be greatly appreciated.

Answer: Thank you for good question. As you have been taught kinetic energy of an object is:

$$E_{kin} = \frac{mv^2}{2}$$

Where m is mass of an object and v is object's velocity. This equation can be used only in case of small velocities : $v \ll c$,where is speed of light. In generalcase the work expended accelerating an object from rest to a relativistic speed is:

$$E_{kin} = \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - mc^2$$

The equation shows that the energy of an object approaches infinity as the velocity *v* approaches the speed of light *c*, thus it is impossible to accelerate an object across this boundary.

The mathematical by-product of this calculation is the mass-energy equivalence formula—the body at rest must have energy content equal to:

$$E_{rest} = E_0 = mc^2$$

At a low speed (v<<c), the relativistic kinetic energy may be approximated well by the classical kinetic energy. This is done bybinomial approximation. Indeed, taking Taylor expansion for the reciprocal square root and keeping first two terms we get:

$$E_{kin} \approx \ mc^2\left(1+\frac{\frac{1}{2}v^2}{c^2}\right) - mc^2 = \frac{1}{2}mv^2$$

So, the total energy E can be partitioned into the energy of the rest mass plus the traditional Newtonian kinetic energy at low speeds.

Visible light, as with all types of electromagnetic radiation (EMR), is emitted and absorbed in tiny "packets" called photons, and exhibits properties of both waves and particles. Important thing is that photon is massless - mass of photon is zero. In empty space, the photon moves at *c* (the speed of light) and its energy and momentum are related by $E = pc$, where p is themagnitude of the momentum vector $\boldsymbol{p}$. This derives from the following relativistic relation, with $m = 0$:

$$E^2 = p^2c^2 + m^2c^4$$

The energy of a photon depend only on its frequency (ν) or inversely, its wavelength (λ):

$$E = \hbar\omega \ = h\nu = \frac{hc}{\lambda}$$

$\omega = 2\pi\nu$ is the angular frequency, and $\hbar = h/2\pi$ is thereduced Planck constant.

References:

1. http://en.wikipedia.org/wiki/Kinetic_energy
2. http://en.wikipedia.org/wiki/Mass_in_special_relativity
3. http://youtu.be/FJ2SVPahBzg
4. http://www.speed-light.info/speed_of_light_variable.htm
5. http://physics.uoregon.edu/~soper/Light/photons.html