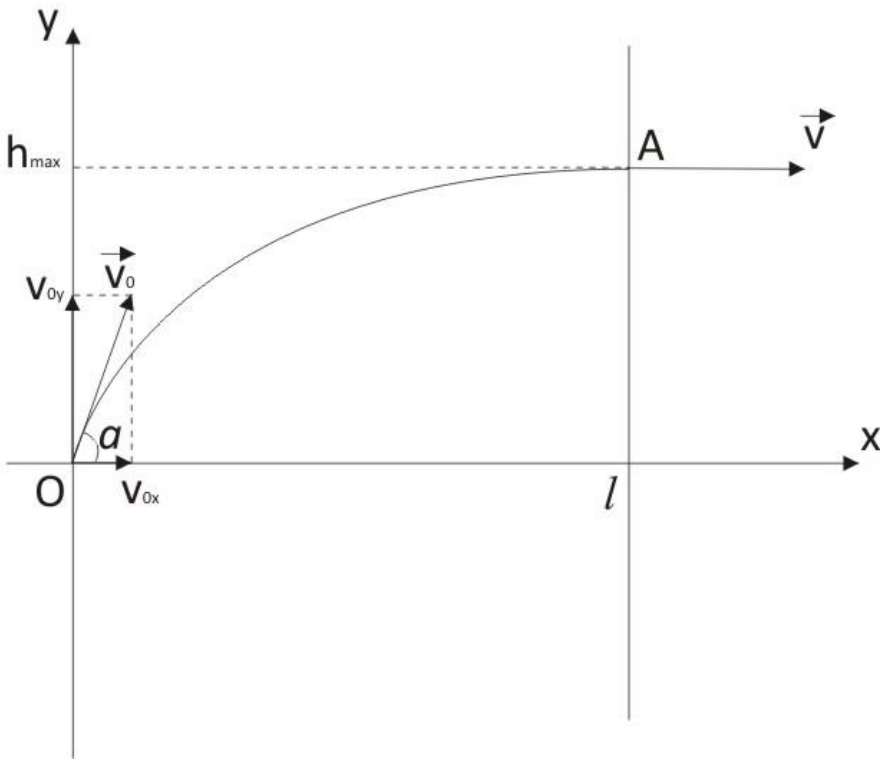
A boy wants to throw a letter wrapped over a stone to his friend across the street 40m wide. The boy's window is 10m below friend's window. How should he throw it?

Solution.

$$l = 40m, h_{max} = 10m, g = 9.8\frac{m}{s^2};$$

$$v_0 - ?\ \alpha - ?$$



A boy should throw a stone so that it will fly in a parabola.

A point *"O"* is the window of the boy;

A point *"A"* is the window of the boy's friend.

A Point "A" is the highest point the stone attained.

A boy has to throw a stone at an angle α, with an initial speed v_0, as in the diagram.

Projections of the speed by the axis:

$$v_x = v_0 cos\alpha;$$

$$v_y = v_0 sin\alpha.$$

We obtain the time of the flight of the stone.

The equation of the projection of the speed for the y-axis:

$$v_y = v_{0y} - gt;$$

$$v_y = v_0 sin\alpha - gt.$$

At the point *"A"* $v_y = 0$

$$0 = v_0 sin\alpha - gt;$$

$$v_0 sin\alpha = gt;$$

$$t = \frac{v_0 sin\alpha}{g}.$$

The max height attained by the stone.

$$h_{max} = v_{0y} t - \frac{gt^2}{2};$$

$$h_{max} = v_0 sin\alpha t - \frac{gt^2}{2};$$

$$t = \frac{v_0 sin\alpha}{g}.$$

$$h_{max} = v_0 sin\alpha \frac{v_0 sin\alpha}{g} - \frac{g}{2}\frac{v_0^2 sin^2\alpha}{g^2};$$

$$h_{max} = \frac{v_0^2 sin^2\alpha}{g} - \frac{v_0^2 sin^2\alpha}{2g};$$

$$h_{max} = \frac{v_0^2 sin^2\alpha}{2g}.$$

The range of the stone:

$$l = v_{0x} t;$$

$$l = v_0 cos\alpha t;$$

$$l = v_0 cos\alpha \frac{v_0 sin\alpha}{g};$$

$$l = \frac{v_0^2 sin\alpha cos\alpha}{g}.$$

We have two equations:

1. $h_{max} = \frac{v_0^2 sin^2\alpha}{2g}$;

2. $l = \frac{v_0^2 sin\alpha cos\alpha}{g}$.

Divide the second equation by the first and obtain the angle α:

$$\frac{l}{h_{max}} = \frac{v_0^2 sin\alpha cos\alpha 2g}{g v_0^2 sin^2\alpha};$$

$$\frac{l}{h_{max}} = \frac{2cos\alpha}{sin\alpha};$$

$$\frac{l}{h_{max}} = \frac{2}{tan\alpha};$$

$$tan\alpha = \frac{2h_{max}}{l}.$$

$$tan\alpha = \frac{2 \cdot 10}{40} = 0.5$$

$$\alpha = 26{,}565°.$$

From the first equation we obtain an initial speed.

$$h_{max} = \frac{v_0^2 sin^2\alpha}{2g};$$

$$v_0 = \frac{\sqrt{2gh_{max}}}{sin\alpha}.$$

$$v_0 = \frac{\sqrt{2 \cdot 9.8 \cdot 10}}{sin(26{,}565°)} = 31.3\left(\frac{m}{s}\right).$$

Answer: A boy should throw a letter wrapped over a stone at an angle $\alpha = 26{,}565°$ at a initial speed of $v_0 = 31.3\frac{m}{s}$.

If the boy is throwing a stone at a lower speed, he will have to increase the angle of throwing.