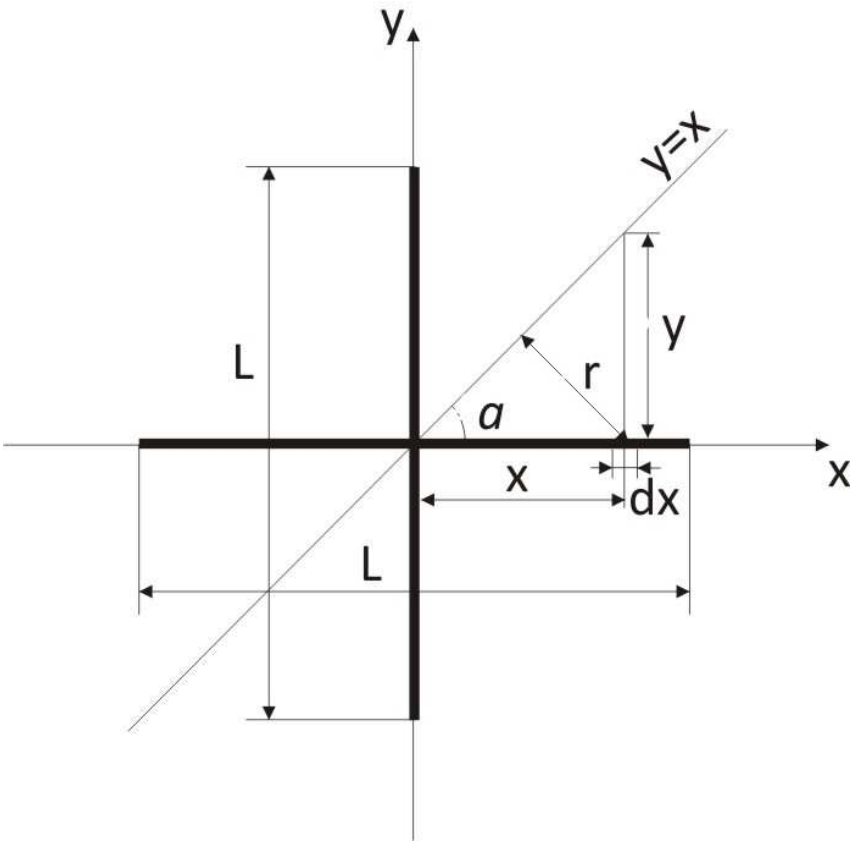
Two rods of equal mass m and length L lie along the x axis and y axis with their centers at the origin. What is the moment of inertia of both about the line x=y?

**Solution.**

$$m, L;$$

$$I - ?$$




The moment of inertia of both rods about the line $y = x$:

$$I = 2I_0;$$

$I_0$ – the moment of inertia of one rod about the line $y = x$.

The mass of the particle of the rod with the length $dx$:

$$dm = \frac{m}{L}dx;$$

The moment of inertia of the particle of the rod with the length $dx$ about the line $y = x$:

$$dI_0 = dmr^2;$$

$$r^2 = x^2 sin^2\alpha;$$

$$dI_0 = \frac{m}{L}dx x^2 sin^2\alpha.$$

The moment of inertia of one rod about the line $y = x$:

$$I_0 = 2\int_0^{\frac{m}{2}} dmr^2 = \frac{2m}{L}sin^2\alpha \int_0^{\frac{L}{2}} x^2 dx = \frac{2m}{L}sin^2(\alpha)\frac{1}{3}\left(\frac{L}{2}\right)^3 = \frac{1}{12}mL^2sin^2\alpha.$$

The moment of inertia of both rods about the line $y = x$:

$$I = 2\frac{1}{12}mL^2sin^2\alpha = \frac{1}{6}mL^2sin^2\alpha.$$

By the diagram:

$$y = x;$$

$$\frac{y}{x} = 1;$$

$$\frac{y}{x} = tan\alpha;$$

$$tan\alpha = 1;$$

$$\alpha = 45°.$$

$$I = \frac{1}{6}mL^2sin^2(45°).$$

**Answer:** The moment of inertia of both rods about the line $y = x$ is $I = \frac{1}{6}mL^2sin^2(45°)$.