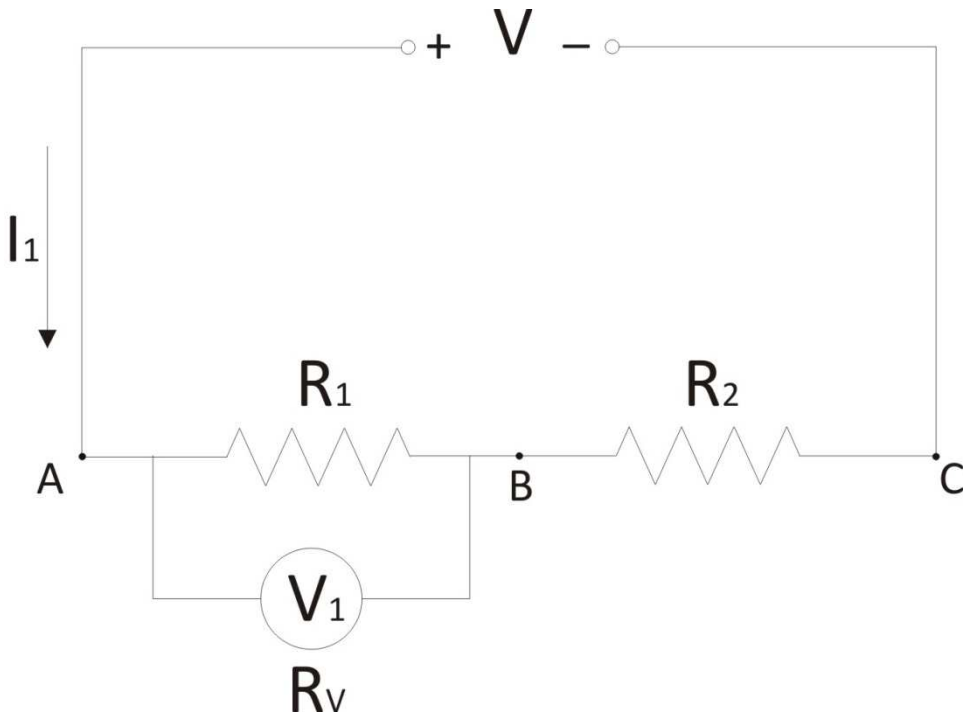
The resistance of 500 ohm and 3000 ohm are placed in series with a 60 V supply. What will be the reading on a voltmeter of internal resistance of 2000 ohm when placed across-
1) 500 ohm resistors;
2) 3000 ohm resistors.

Solution.

$$R_1 = 500Ohm, R_2 = 3000Ohm, V = 60V, R_V = 2000Ohm;$$
$$V_1 - ? V_2 - ?$$

1)



The resistance between the points *A* and *B:*

$$\frac{1}{R_{AB}} = \frac{1}{R_1} + \frac{1}{R_V};$$

R_V – the internal resistance of the voltmeter.

$$R_{AB} = \frac{R_1 R_V}{R_1 + R_V}.$$

The resistance between the points *A* and *C:*

$$R_{AC} = R_{AB} + R_2;$$

$$R_{AC} = \frac{R_1 R_V}{R_1 + R_V} + R_2;$$

$$R_{AC} = \frac{R_1 R_V + R_1 R_2 + R_V R_2}{R_1 + R_V}.$$

By Ohm's law the amperage I_1 is:

$$I_1 = \frac{V}{R_{AC}};$$

$$I_1 = \frac{V(R_1 + R_V)}{R_1R_V + R_1R_2 + R_VR_2}.$$

The voltage across resistance R_1 is the same as the voltage across voltmeter and the same as the voltage across the points *A* and *B* then:

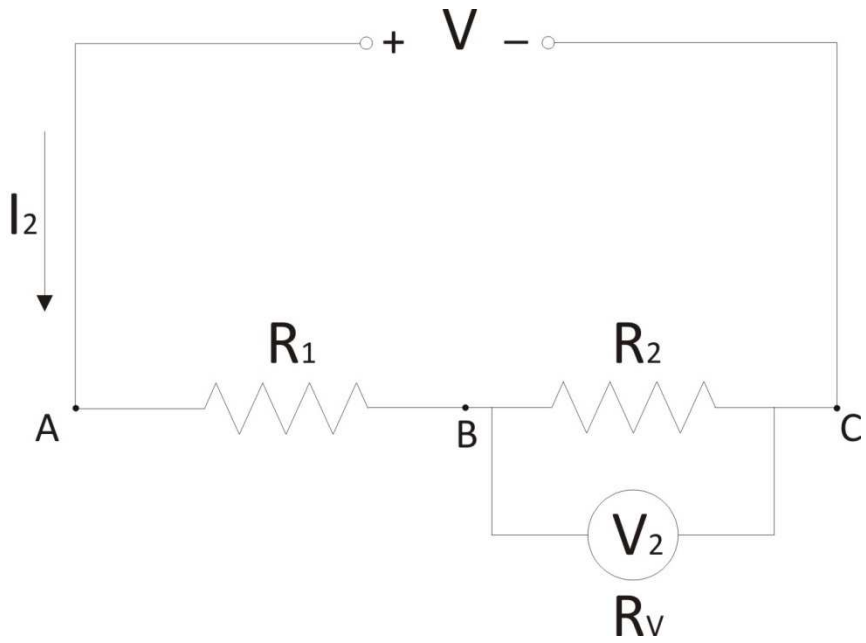
$$V_1 = I_1R_{AB};$$

$$V_1 = \frac{V(R_1 + R_V)}{R_1R_V + R_1R_2 + R_VR_2} \cdot \frac{R_1R_V}{R_1+R_V};$$

$$V_1 = \frac{VR_1R_V}{R_1R_V + R_1R_2 + R_VR_2}.$$

$$V_1 = \frac{60 \cdot 500 \cdot 2000}{500 \cdot 2000 + 500 \cdot 3000 + 2000 \cdot 3000} = 7.06(V).$$

2)



The resistance between the points *B* and *C:*

$$\frac{1}{R_{BC}} = \frac{1}{R_2} + \frac{1}{R_V};$$

R_V – the internal resistance of the voltmeter.

$$R_{BC} = \frac{R_2R_V}{R_2+R_V}.$$

The resistance between the points *A* and *C:*

$$R_{AC} = R_1 + R_{BC};$$

$$R_{AC} = R_1 + \frac{R_2 R_V}{R_2 + R_V};$$

$$R_{AC} = \frac{R_1 R_2 + R_1 R_V + R_2 R_V}{R_2 + R_V}.$$

By Ohm's law the amperage I_2 is:

$$I_2 = \frac{V}{R_{AC}};$$

$$I_2 = \frac{V(R_2 + R_V)}{R_1 R_2 + R_1 R_V + R_2 R_V}.$$

The voltage across resistance R_2 is the same as the voltage across voltmeter and the same as the voltage across the points *B* and *C* then:

$$V_2 = I_2 R_{BC};$$

$$V_2 = \frac{V(R_2 + R_V)}{R_1 R_2 + R_1 R_V + R_2 R_V} \cdot \frac{R_2 R_V}{R_2 + R_V};$$

$$V_2 = \frac{V R_2 R_V}{R_1 R_V + R_1 R_2 + R_V R_2}.$$

$$V_2 = \frac{60 \cdot 3000 \cdot 2000}{500 \cdot 2000 + 500 \cdot 3000 + 2000 \cdot 3000} = 42.35(V).$$

Answer:

1) $V_1 = 7.06V$;

2) $V_2 = 42.35V$.