

What do you understand by the normal modes of coupled oscillators? If a coupled system has many normal modes, do all normal modes have the same frequency? Calculate the velocity of elastic longitudinal wave along a stretched steel wire, given density of steel is $8000 \text{ kg}\cdot\text{m}^{-3}$, Young's modulus of elasticity is $2\cdot10^{11} \text{ N}\cdot\text{m}^{-2}$.

Answer: A normal mode of an oscillating system is a pattern of motion in which all parts of the system move sinusoidally with the same frequency and with a fixed phase relation. Normal mode is a characteristic of a whole system. The most general motion of a coupled oscillators system is a superposition of its normal modes. Even though uncoupled angular frequencies need not necessarily be the same, for the normal mode the effect of coupling is that all bodies oscillate with the same frequency. For a system of N coupled oscillators there exist N normal modes in which all oscillators move with the same frequency and thus have fixed amplitude ratios.

Velocity of longitudinal waves in solid materials can be calculated by the next formula:

$$v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2\cdot10^{11}}{8000}} = 5000 \text{ m/s};$$