Discuss the principle of superposition. Two collinear SHMs, with amplitudes 5 cm and 12 cm are superposed. Calculate the resultant amplitude when the SHMs differ in phase by I) 60°; II) 90°; III) 120°.

Solution: When a body is a subject to more than one harmonic force, each trying to move the body in its own direction with SHM, we say that there is a superposition of SHMs. The superposition of two simple harmonic motions that produce a displacement of the body along the same line in the case when both have the same frequency can be described as the sum of equations of the body displacement:

$$x_1 = A_1 \cdot \sin(\omega t + \varphi_1);\ \ x_2 = A_2 \cdot \sin(\omega t + \varphi_2);\ \ x = x_1 + x_2 = A_1 \cdot \sin(\omega t + \varphi_1) + A_2 \cdot \sin(\omega t + \varphi_2);$$

If $\varphi_1 = \varphi_2 = \varphi$, we say that the two motions are in phase. Then the resultant motion is:

$$x = A_1 \cdot \sin(\omega t + \varphi) + A_2 \cdot \sin(\omega t + \varphi) = (A_1 + A_2) \cdot \sin(\omega t + \varphi);$$

Equation shows that the resultant motion is also SHM with the same angular frequency. The motion has an amplitude equal to the sum of the amplitudes of the two motions; that is, $A = A_1 + A_2$;

When $\varphi_2 = \varphi_1 + \pi$, we have: $x_2 = A_2 \cdot \sin(\omega t + \varphi_2) = A_2 \cdot \sin(\omega t + \varphi_1 + \pi) = -A_2 \cdot \sin(\omega t + \varphi_1);$

Then the resultant motion is $x = A_1 \cdot \sin(\omega t + \varphi) - A_2 \cdot \sin(\omega t + \varphi) = (A_1 - A_2) \cdot \sin(\omega t + \varphi);$

It shows that the resultant motion is SHM with the same angular frequency and an amplitude equal to the difference of the amplitude of the two motions; that is, $A = A_1 - A_2$; we say that the motions are in opposition.

In the general case, where the phase difference is arbitrary, the resultant motion is also SHM with the same angular frequency and amplitude given by $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cdot \cos(\varphi_2 - \varphi_1)};$

I) $A = \sqrt{0.05^2 + 0.12^2 + 2 \cdot 0.05 \cdot 0.12 \cdot \cos 60^\circ} = 0.151$ m;

II) $A = \sqrt{0.05^2 + 0.12^2 + 2 \cdot 0.05 \cdot 0.12 \cdot \cos 90^\circ} = 0.130$ m;

III) $A = \sqrt{0.05^2 + 0.12^2 + 2 \cdot 0.05 \cdot 0.12 \cdot \cos 120^\circ} = 0.104$ m;