

Write the equation of motion of a simple harmonic oscillator which has amplitude of 5 cm and it executes 150 oscillations in 5 minutes with an initial phase of 45° . Also obtain the value of its maximum velocity.

Solution: As it is known, general equation of simple harmonic oscillations is: $x = A \cdot \sin(\omega \cdot t + \varphi_0)$, where x – displacement of the oscillating body from the equilibrium position, m; A – amplitude of oscillations, m; ω – angular frequency, rad/s; φ_0 is the phase of oscillations.

According to the problem conditions, $A = 5\text{cm} = 0.05\text{m}$; $\omega = 2\pi \cdot f = 2\pi \cdot \frac{N}{t} = 2 \cdot 3.14 \cdot \frac{150}{5 \cdot 60} = 3.14 \text{ rad/s}$;

(f is the frequency of oscillations, Hz). Then, $x = 0.05 \cdot \sin(3.14 \cdot t + 45^\circ)$.

Velocity of harmonically oscillating body can be calculated as the derivative of its movement by time:

$$v = \frac{dx}{dt} = [0.05 \cdot \sin(3.14 \cdot t + 45^\circ)]' = 3.14 \cdot 0.05 \cdot \cos(3.14 \cdot t + 45^\circ) = 0.157 \cdot \cos(3.14 \cdot t + 45^\circ).$$

Numerical factor at the front is the maximum velocity of harmonic oscillator; it is equal to 0.157 m/s.

Answer: $x = 0.05 \cdot \sin(3.14 \cdot t + 45^\circ)$; 0.157 m/s.