An experimentalist observed the motion of soot particles of radius $0.5 \cdot 10^{-4}$ cm in water-glycerine solution characterized by η = $2.80 \cdot 10^{-3}$ kg·m^{-1}·s^{-1} at 300 K for 10 s. The observed value of $\overline{x^2}$ was $3.30 \cdot 10^{-8}$ cm^2. Calculate Boltzmann constant and hence Avogadro's number.

Solution: According to the Einstein's theory of Brownian motion of molecules, main equation of which is: $\frac{\overline{x^2}}{2t} = \frac{k_B \cdot T}{6\pi \cdot \eta \cdot r}$, then $k_B = \frac{3\pi \cdot \eta \cdot r \cdot \overline{x^2}}{t \cdot T} = \frac{3 \cdot 3.14 \cdot 2.8 \cdot 10^{-3} \cdot 5 \cdot 10^{-7} \cdot 3.3 \cdot 10^{-12}}{10 \cdot 300} = 1.45 \cdot 10^{-23}$ $\frac{\text{J}}{\text{K}}$; (all values for calculation were converted to the main SI units).

Avogadro's number can be calculated as: $N_A = \frac{\overline{R}}{k_B} = \frac{8.314}{1.45 \cdot 10^{-23}} = 5.73 \cdot 10^{23}$ mol^{-1}; ($\overline{R}$ is the universal gas constant, 8.314 J·mol^{-1}·K^{-1}).

Answer: $k_B = 1.45 \cdot 10^{-23}$ $\frac{\text{J}}{\text{K}}$; $N_A = 5.73 \cdot 10^{23}$ mol^{-1}.