

Calculate the work done by one mole of a Van der Waals' gas if during its isothermal expansion its volume increases from 1 m^3 to 2 m^3 at a temperature 300 K. Take $a=1.39 \cdot 10^{-6} \text{ atm} \cdot \text{m}^6 \cdot \text{mol}^{-2}$ and $b = 39.1 \cdot 10^{-6} \text{ m}^3 \cdot \text{mol}^{-1}$.

Solution: As you know, work of 1 mol of gas can be calculated as: $W = \int_{v_1}^{v_2} pdv$; where v is the molar volume, m^3/mol . The Van der Waals equation of state for 1 mole of gas is: $\left(p + \frac{a}{v^2} \right) \cdot (v - b) = R_0 T$;

From this equation we obtain that $p = \frac{R_0 T}{v - b} - \frac{a}{v^2}$;

$$\text{Then, } W = \int_{v_1}^{v_2} \left(\frac{R_0 T}{v - b} - \frac{a}{v^2} \right) dv = \int_{v_1}^{v_2} \frac{R_0 T}{v - b} dv - \int_{v_1}^{v_2} \frac{a}{v^2} dv = R_0 T \cdot \ln \frac{v_2 - b}{v_1 - b} + a \cdot \left(\frac{1}{v_2} - \frac{1}{v_1} \right);$$

Where $v = \frac{V}{n}$; $v_1 = \frac{1}{1} = 1 \frac{\text{m}^3}{\text{mol}}$; $v_2 = \frac{2}{1} = 2 \frac{\text{m}^3}{\text{mol}}$; also, we must convert the value of coefficient a into the SI units: $a = 1.39 \cdot 10^{-6} \cdot 101,325 \text{ N} \cdot \text{m}^{-2} \cdot \text{m}^6 \cdot \text{mol}^{-2} = 0.141 \text{ N} \cdot \text{m}^4 \cdot \text{mol}^{-2}$.

$$W = 8.314 \cdot 300 \cdot \ln \frac{2 - 39.1 \cdot 10^{-6}}{1 - 39.1 \cdot 10^{-6}} + 0.141 \cdot \left(\frac{1}{2} - \frac{1}{1} \right) = 1728.8 \text{ J};$$

Answer: 1728.8 J.