The displacement of a particle is zero at t=0 seconds and it is x at t seconds. It stars in a positive x direction with a velocity that varies v=k√x where k is a constant. Show that velocity varies with time.

By definition:

$v = \frac{dx}{dt}$

x – displacement

t – time

And:

$v = k\sqrt{x} \quad => x = \left(\frac{v}{k}\right)^2$

or

$dv = \frac{k}{2\sqrt{x}}dx$

Therefore:

$dx = \frac{2\sqrt{x}}{k}dv = \left| x = \left(\frac{v}{k}\right)^2 \right| = \frac{2v}{k^2}dv$

Substitute to $v = \frac{dx}{dt}$:

$v = \frac{\frac{2v}{k^2}dv}{dt}$

or:

$\frac{\frac{2v}{k^2}dv}{v} = dt$

$\frac{2}{k^2}dv = dt$

Integrate:

$\frac{2v}{k^2} + C = t$

C – some constant

displacement of a particle is zero at t=0 seconds and $v = k\sqrt{x}$ therefore v=0 then t=0:

$C = 0$

$\frac{2v}{k^2} = t$

$v = \frac{k^2}{2}t$

Answer: $v = \frac{k^2}{2}t$