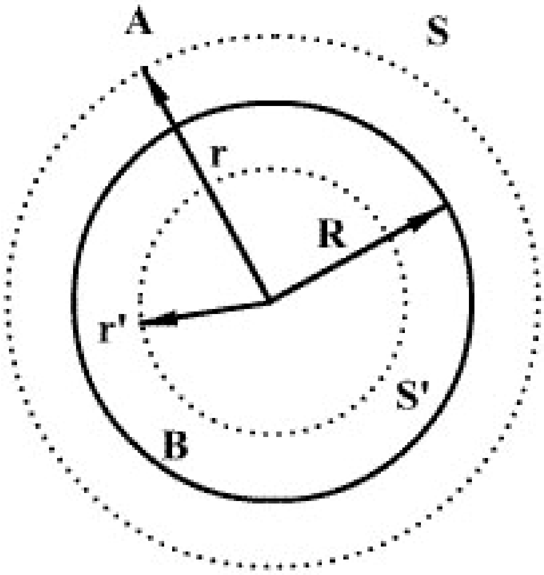
Assuming that the charge 'q' is uniformly distributed in a spherical volume of radius 'R'. Discuss the variation of a) electric intensity b) potential as the field point is moved from the center of the sphere to infinity.

Solution.



a)

We conclude our spherical volume of a symmetrical surface S of radius $r > R$. Electric flux through the surface S is equal to:

$$\Phi_E = \oiint_S EdS = E4\pi r^2;$$

By Gauss's law:

$$\Phi_E = \frac{q}{\varepsilon_0};$$

Therefore:

$$E4\pi r^2 = \frac{q}{\varepsilon_0};$$

The electric field intensity outside the spherical volume $r > R$:

$$E = \frac{q}{4\pi r^2 \varepsilon_0}.$$

At the point 'B', inside of the spherical volume $r < R$ electric field is determined by only a charge q' inside the sphere of a radius of r'. Electric flux through the surface S' is equal to:

$$\Phi_E = \oiint_{S'} EdS = E4\pi r'^2;$$

By Gauss's law:

$$\Phi_E = \frac{q'}{\varepsilon_0};$$

$$q' = \rho_q V';$$

The volume charge density:

$$\rho_q = \frac{q}{V};$$

$$V = \frac{4}{3}\pi R^3;$$

$$\rho_q = \frac{3q}{4\pi R^3}.$$

$$q' = \frac{3q}{4\pi R^3} V';$$

$$V' = \frac{4}{3}\pi r'^3;$$

$$q' = \frac{3q}{4\pi R^3} \cdot \frac{4}{3}\pi r'^3 = \frac{qr'^3}{R^3}.$$

$$\Phi_E = \frac{qr'^3}{\varepsilon_0 R^3}.$$

$$E 4\pi r'^2 = \frac{qr'^3}{\varepsilon_0 R^3};$$

$$E = \frac{qr'}{4\pi\varepsilon_0 R^3}.$$

The electric field intensity inside the spherical volume $r < R$:

$$E = \frac{qr}{4\pi\varepsilon_0 R^3}.$$

b)

The electric potential outside the spherical volume $r > R$:

$$dV_E = -Edr = \frac{q}{4\pi\varepsilon_0 r^2} dr;$$

$$V_E = -E \int_{\infty}^{r} dr = -\frac{q}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{dr}{r^2};$$

$$V_E = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right).$$

$$V_E = \frac{q}{4\pi\varepsilon_0 r}.$$

The electric potential inside the spherical volume $r < R$:

$$V_E = -E\int_{\infty}^{r} dr = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0 r^2} dr - \int_{R}^{r} \frac{q}{4\pi\varepsilon_0 R^3} r dr\,;$$

$$-\int_{R}^{r} \frac{q}{4\pi\varepsilon_0 R^3} r dr = -\frac{q}{4\pi\varepsilon_0 R^3} \cdot \frac{r^2}{2} + \frac{q}{4\pi\varepsilon_0 R^3} \cdot \frac{R^2}{2};$$

$$V_E = \frac{3q}{8\pi\varepsilon_0 R} - \frac{qr^2}{8\pi\varepsilon_0 R^3}.$$

Answer:

a)

The electric field intensity inside the spherical volume $r < R$:

$$E = \frac{qr}{4\pi\varepsilon_0 R^3}.$$

The electric field intensity outside the spherical volume $r > R$:

$$E = \frac{q}{4\pi r^2 \varepsilon_0}.$$

b)

The electric potential inside the spherical volume $r < R$:

$$V_E = \frac{3q}{8\pi\varepsilon_0 R} - \frac{qr^2}{8\pi\varepsilon_0 R^3}.$$

The electric potential outside the spherical volume $r > R$:

$$V_E = \frac{q}{4\pi\varepsilon_0 r}.$$