A projectile is fired vertically from Earth's surface with an initial speed v0. Neglecting air drag, how far above the surface of Earth will it go? (Use any variable or symbol stated above along with the following as necessary: ME for the mass of the Earth, rE for the radius of the Earth, and G for the gravitational constant.)

v_0 - initial speed
M_E - mass of the Earth
r_E - radius of the Earth
G - gravitational constant
The law of conservation of energy:
$T + U = const$
$T = \frac{mv^2}{2}$ - kinetic energy
m - mass of the body
v - speed
$U = -\frac{GMm}{r}$ - potential energy
The law of conservation of energy in another form:
$T_1 + U_1 = T_2 + U_2$
1 - initial state
2 - final state
$T_1 = \frac{mv_0^2}{2}$ - initial kinetic energy
$U_1 = -\frac{GM_E m}{r_E}$ – initial potential energy
If $r = r_{max}$: $T_2 = 0$ and $U_2 = -\frac{GM_E m}{r_{max}}$
Substitute to the law of conservation of energy:
$\frac{mv_0^2}{2} - \frac{GM_E m}{r_E} = 0 - \frac{GM_E m}{r_{max}}$
$r_{max} = -\frac{GM_E}{\frac{v_0^2}{2} - \frac{GM_E}{r_E}} = \frac{r_E}{1 - \frac{v_0^2 r_E}{GM_E}}$
Distance above the surface of Earth equals:
$r = r_{max} - r_E = \frac{r_E}{1 - \frac{v_0^2 r_E}{GM_E}} - r_E = r_E \left(\frac{\frac{v_0^2 r_E}{GM_E}}{1 - \frac{v_0^2 r_E}{GM_E}} \right) = \frac{r_E}{\frac{GM_E}{v_0^2 r_E} - 1}$
If $1 - \frac{v_0^2 r_E}{GM_E} > 0$
Finally:
if $v_0^2 \geq GM_E / r_E \quad r = \infty$
Answer: if $v_0^2 < GM_E / r_E \quad r = \frac{r_E}{\frac{GM_E}{v_0^2 r_E} - 1}$, if $v_0^2 \geq GM_E / r_E \quad r = \infty$