An undersea research chamber is spherical with an external diameter of 5.20 m. The mass of the chamber is 74 400 kg. It is anchored to the sea bottom by a cable. What is (a) the buoyant force on the chamber and (b) the tension in the cable? (Given density of seawater = 1025 kg/m^3).

Solution.

$$d = 5.2m, m = 74400kg, \rho_w = 1025\frac{kg}{m^3}, g = 9.8\frac{m}{s^2};$$

$$B-?\, F-?$$

(a)

The buoyant force:

$$B = \rho_w gV;$$

ρ_w - the density of seawater;
g – the gravity acceleration.

The volume of the chamber:

$$V = \frac{4}{3}\pi r^3;$$

The radius of the chamber:

$$r = \frac{d}{2};$$

d – the diameter the chamber.

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\frac{d^3}{8} = \frac{1}{6}\pi d^3;$$

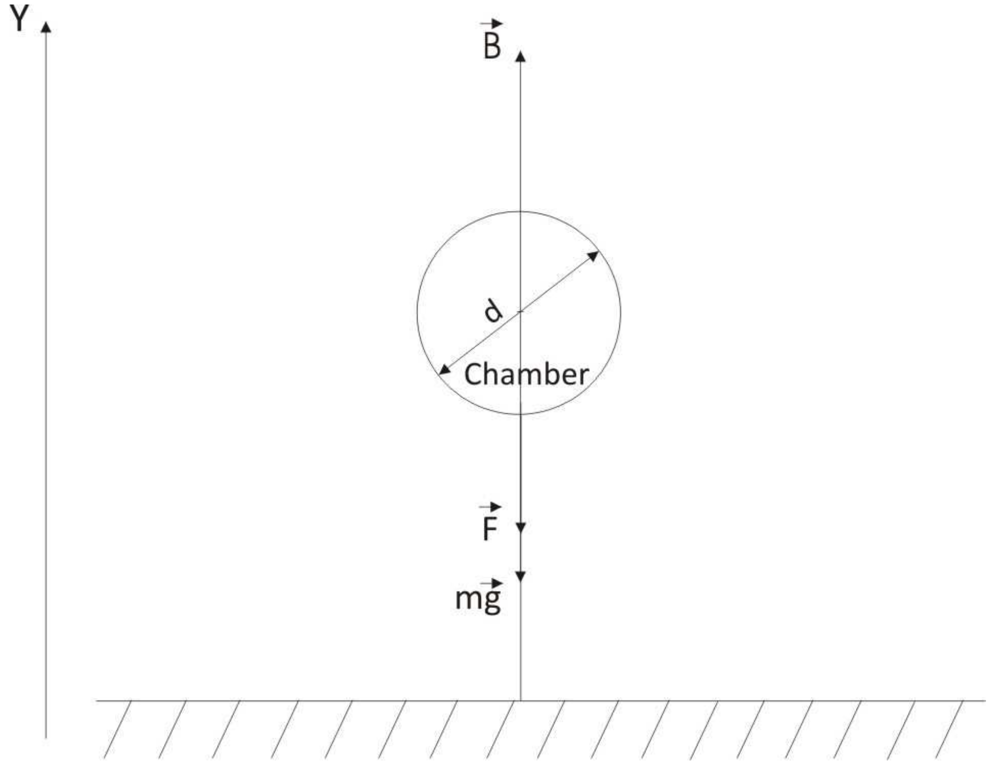
$$V = \frac{1}{6}\pi d^3.$$

Finally for the buoyant force:

$$B = \frac{1}{6}\pi d^3 \rho_w g.$$

$$B = \frac{1}{6}\cdot 3.14 \cdot (5.2)^3 \cdot 1025 \cdot 9.8 \approx 739160(N).$$

(b)



Newton's second law in vector form:

$$m\vec{a} = \vec{B} + \vec{F} + m\vec{g}.$$

F - the tension in the cable;
B - the buoyant force;
m - the mass of the chamber;
g – the gravity acceleration.

A chamber is at rest, then:

$$\vec{a} = 0.$$

$$0 = \vec{B} + \vec{F} + m\vec{g}.$$

Newton's second law in projection on Y:

$$0 = B - F - mg;$$

$$F = B - mg.$$

The buoyant force from the first part of problem **(a):**

$$B = \frac{1}{6}\pi d^3 \rho_w g.$$

$$F = \frac{1}{6}\pi d^3 \rho_w g - mg.$$

Finally for the tension in the cable:

$$F = \left(\frac{1}{6}\pi d^3 \rho_w - m\right) g.$$

$$F = \left(\frac{1}{6} \cdot 3.14 \cdot (5.2)^3 \cdot 1025 - 74400\right) \cdot 9.8 \approx 10040(N).$$

Answer:

(a) the buoyant force is $B = 739160N$;

(b) tension in the cable is $F = 10040N$.