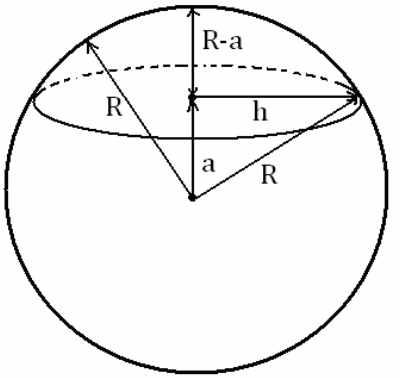
QUESTION

A sphere of radius R is centered at the origin. A plane intersects this sphere at a distance a from its center such that a<R. determine the volume of the smaller of the regions bound by the sphere and the plane . (you may use Cartesian coordinates or spherical polar coordinates.)

SOLUTION:

The region bound by the sphere and the plane called a spherical cap



The volume of the spherical cap is

$$V = \frac{\pi(R-a)}{6}(3h^2 + (R-a)^2)$$

Or

$$V = \frac{\pi(R-a)^2}{3}(3R - (R-a)) = \frac{\pi(R-a)^2}{3}(2R + a)$$

ANSWER

$$V = \frac{\pi(R-a)}{6}(3h^2 + (R-a)^2) = \frac{\pi(R-a)^2}{3}(2R + a)$$