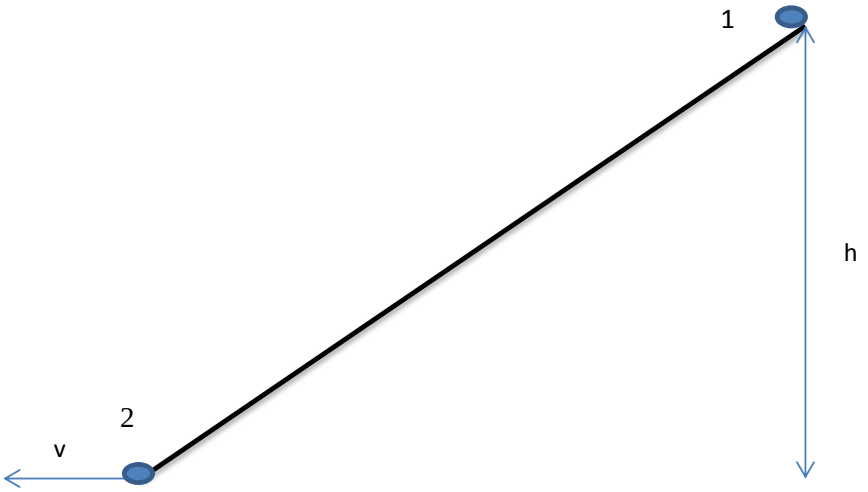
Suppose you are designing a cart for carrying goods downhill. To maximize the cart speed, should you design the wheels so that their moments of inertia about their rotation axes are large or small, or it does not matter? Explain assuming that the mechanical energy is conserved

The conservation law of mechanical energy:

$T + U = const$

or:

$T_1 + U_1 = T_2 + U_2$



For state (1): $T_1 = 0, U_1 = mgh$

For state (2): $U_2 = 0, T_2 = \frac{mv^2}{2} + \frac{Iw^2}{2}$

m – mass of cart;

v – cart's speed;

I – sum of moments of inertia all wheels;

w – angular velocity of wheels.

$w = v/r$, where r – radius of wheel.

Therefore:

$$mgh = \frac{mv^2}{2} + \frac{I\left(\frac{v}{r}\right)^2}{2} = \frac{mv^2}{2}\left(1 + \frac{I}{mr^2}\right)$$

$$v = \sqrt{\frac{2gh}{\left(1+\frac{I}{mr^2}\right)}}$$

Obviously, to maximize the cart speed, you should design the wheels so that their moments of inertia about their rotation axes are small as it possible. In ideal case $I \ll mr^2$ and:

$v_{max} = \sqrt{2gh}$