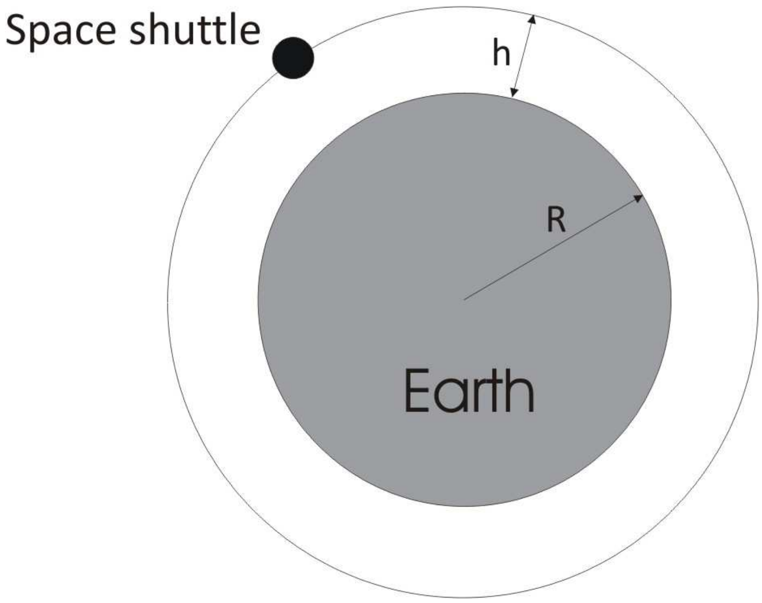
Obtain an expression for the time period of a satellite orbiting the earth. A space shuttle is in a circular orbit at a height of 250 km from the earth's surface, where the acceleration due to earth's gravity is 0.93 g. Calculate the period of its orbit. Take g = 9.8 ms^{-2} and the radius of the earth R = 6.37 × 10^6m.

Solution.

$$g = 9.8\frac{m}{s^2}, g^* = 0.93g, R = 6.37 \cdot 10^6 m, h = 250km = 250 \cdot 10^3 m;$$

$$T-?$$



The time period of a satellite orbiting the earth:

$$T = \frac{2\pi(R+h)}{v};$$

R - the radius of the earth;

h - the height of the circular orbit;

v - the speed of the space shuttle in a circular orbit.

Newton's second law:

$$ma = mg^*;$$

m - the mass of the space shuttle.

g^* - the acceleration due to earth's gravity at a height of 250 km.

$$a = g^*;$$
$$g^* = 0.93g;$$
$$a = 0.93g;$$

a - the radial acceleration.

$$a = \frac{v^2}{(R+h)};$$
$$\frac{v^2}{(R+h)} = 0.93g;$$
$$v = \sqrt{0.93g(R+h)};$$

$$T = \frac{2\pi(R+h)}{\sqrt{0.93g(R+h)}} = \frac{2\pi}{\sqrt{0.93g}} \cdot \frac{R+h}{\sqrt{R+h}} = 2\pi \cdot \frac{\sqrt{R+h}}{\sqrt{0.93g}} = 2\pi\sqrt{\frac{R+h}{0.93g}};$$

$$T = 2\pi\sqrt{\frac{R+h}{0.93g}}.$$

$$T = 2 \cdot 3.14\sqrt{\frac{6.37 \cdot 10^6 + 250 \cdot 10^3}{0.93 \cdot 9.8}} = 5352(s).$$

Answer: $T = 5352s$.