A circular disc rotates on a thin air film with a period of 0.3 s. Its moment of inertia about its axis of rotation is 0.06 kg m^2. A small mass is dropped onto the disc and rotates with it. The moment of inertia of the mass about the axis of rotation is 0.04 kg m^2. Determine the final period of the rotating disc and mass.

Solution.

$$T_1 = 0.3s, I_1 = 0.06kg \cdot m^2, I = 0.04kg \cdot m^2;$$
$$T_1 - ?$$

The angular momentum of the disc is:

$$L_1 = I_1\omega_1;$$

I_1 - the moment of inertia of the disc;
ω_1 - the angular velocity of the disc.

$$\omega_1 = \frac{2\pi}{T_1};$$

T_1 – the time period of the disc.

$$L_1 = I_1\frac{2\pi}{T_1};$$

The angular momentum of the disc with the small mass is:

$$L_2 = I_2\frac{2\pi}{T_2};$$
$$I_2 = I_1 + I;$$

I_2 - the moment of inertia of the system: the disc with the small mass.
I – the moment of inertia of the small mass.

$$L_2 = (I_1 + I)\frac{2\pi}{T_2}.$$

The law of conservation of angular momentum acts in this situation, because no external torque acts on a disc then:

$$L_1 = L_2;$$
$$I_1\frac{2\pi}{T_1} = (I_1 + I)\frac{2\pi}{T_2};$$
$$\frac{I_1}{T_1} = \frac{I_1 + I}{T_2};$$
$$T_2 = \frac{I_1 + I}{I_1}T_1;$$
$$T_2 = \left(1 + \frac{I}{I_1}\right)T_1.$$
$$T_2 = \left(1 + \frac{0.06}{0.04}\right)0.3 = 0.75(s).$$

Answer: $T_2 = 0.75s$.