What do you understand by the term inertial force? Do inertial forces arise in inertial frames?

Explain. Derive the expression for the inertial force exerted on an accelerating object in

translational motion in the non-inertial frame attached to it.

A fictitious force, also called a phantom force, pseudo force, d'Alembert force or inertial force, is an apparent force that acts on all masses in a non-inertial frame of reference, such as a rotating reference frame.

The force F does not arise from any physical interaction but rather from the acceleration a of the non-inertial reference frame itself.

A fictitious force arises when a frame of reference is accelerating compared to a non-accelerating frame. As a frame can accelerate in any arbitrary way, so can fictitious forces be as arbitrary (but only in direct response to the acceleration of the frame). However, four fictitious forces are defined for frames accelerated in commonly occurring ways: one caused by any relative acceleration of the origin in a straight line (rectilinear acceleration), two caused by any rotation (centrifugal force and Coriolis force) and a fourth, called the Euler force, caused by a variable rate of rotation, should that occur.

To answer this question, let the coordinate axis in B be represented by unit vectors $\overrightarrow{u_j}$ with j any of { 1, 2, 3 } for the three coordinate axes. Then

$$\overrightarrow{x_B} = \sum_{j=1}^{3} x_j \overrightarrow{u_j}$$

The interpretation of this equation is that $x_B$ is the vector displacement of the particle as expressed in terms of the coordinates in frame B at time t. From frame A the particle is located at:

$$\overrightarrow{x_A} = \overrightarrow{X_{AB}} + \sum_{j=1}^{3} x_j \overrightarrow{u_j}$$

$$\frac{d\overrightarrow{x_A}}{dt} = \overrightarrow{v_{AB}} + \overrightarrow{v_B} + \sum_{j=1}^{3} x_j \frac{d\overrightarrow{u_j}}{dt}$$

$$\frac{d^2\overrightarrow{x_A}}{dt^2} = \overrightarrow{a_{AB}} + \overrightarrow{a_B} + 2\sum_{j=1}^{3} v_j \frac{d\overrightarrow{u_j}}{dt} + \sum_{j=1}^{3} x_j \frac{d^2\overrightarrow{u_j}}{dt^2}$$

$$\overrightarrow{F_A} = \overrightarrow{F_B} + m\overrightarrow{a_{AB}} + 2m\sum_{j=1}^{3} v_j \frac{d\overrightarrow{u_j}}{dt} + m\sum_{j=1}^{3} x_j \frac{d^2\overrightarrow{u_j}}{dt^2}$$