Alpha-Centari, is located at a distance of 4.5 light years. What is the diameter of a reflecting telescope, placed in Earths orbit, observed in visible light at 550nm, in order to resolve:

a-A planet orbiting its star at the same distance as earth is from the sun.

b- A sun sized star.

c- Earth sized planet?

Solution

According the theory of difraction the angular resolution of telescope with a objective diameter ***D*** is

$\alpha = 1.22 * \frac{\lambda}{D} * 206265'$ (in angular seconds)

Where λ is wavelength of light

a) We have the angular distance between star and planet, if a planet orbiting its star at the same distance as earth is from the sun and star is located at a distance of 4.5 light years= 284582 astronomical unit, is

$$\alpha = \frac{1 a.u.}{284582 \text{a.u}} * 206265' \approx 0.725'$$

From hence we have minimal diameter od telescope

$$D_{(a)\min} = \frac{1.22 * \lambda * 206265'}{\alpha} = \frac{1.22 * 550nm * 206265'}{\alpha} = 191mm \Rightarrow$$
$$D_\alpha > 191mm$$

b) We have the angular size of star, if a star is like Sun (star radius is $6{,}9551 \cdot 10^8$ m) and star is located at a distance of 4.5 light years= 284582 astronomical unit $\approx 4.26 \cdot 10^{16}$ m, is

$$\alpha = \frac{6{,}9551 \cdot 10^8 \text{ m}}{4.26 \cdot 10^{16} \text{m}} * 206265' \approx 0.0033'$$

From hence we have minimal diameter od telescope

$$D_{(b)\min} = \frac{1.22 * 550nm * 206265'}{0.0033'} \approx 41962mm \approx 42m \Rightarrow$$
$$D_b > 42m$$

c) We have the angular size of planet, if a planet is like Earth (Earth radius is $6{,}4 \cdot 10^6$ m) and star is located at a distance of 4.5 light years= 284582 astronomical unit $\approx 4.26 \cdot 10^{16}$ m, is

$$\alpha = \frac{6{,}410^{6}\ \mathrm{m}}{4.26\cdot 10^{16}\ \mathrm{m}} * 206265'' \approx 0.00003'$$

From hence we have minimal diameter od telescope

$$D_{(c)\min} = \frac{1.22 * 550nm * 206265''}{0.00003'} \approx 4200m \Rightarrow$$
$$D_c > 4200m$$

Answer

a)

$$D_{(a)\min} = \frac{1.22 * \lambda * 206265''}{\alpha} = \frac{1.22 * 550nm * 206265''}{\alpha} = 191mm \Rightarrow$$
$$D_\alpha > 191mm$$

b)

$$D_{(b)\min} = \frac{1.22 * 550nm * 206265''}{0.0033'} \approx 41962mm \approx 42m \Rightarrow$$
$$D_b > 42m$$

c)

$$D_{(c)\min} = \frac{1.22 * 550nm * 206265''}{0.00003'} \approx 4200m \Rightarrow$$
$$D_c > 4200m$$