QUESTION:

A running man has the half the kinetic energy of a body of half his mass. the man speeds up by 1m/s and has the same kinetic energy as the boy. What were the original speed of the man and the boy?

SOLUTION:

The kinetic energy of a man is $E_{k,man}=\frac{m_{man}\upsilon_{man}^2}{2}$ The kinetic energy of a boy is $E_{k,boy}=\frac{m_{boy}\upsilon_{boy}^2}{2}=\frac{m_{man}}{2}\cdot\frac{\upsilon_{boy}^2}{2}=\frac{m_{man}\upsilon_{boy}^2}{4}$. When man speeds up by a $\Delta\upsilon=1\ m/s$, his kinetic energy is $E'_{k,man}=\frac{m_{man}(\upsilon_{man}+\Delta\upsilon)^2}{2}$

As a man has the half the kinetic energy of a boy:

$$E_{k,man}=\frac{1}{2}E_{k,boy}$$

And after speeding up man has the same kinetic energy as a boy:

$$E'_{k,man}=E_{k,boy}$$

Hence:

$$\begin{cases}\frac{m_{man}\upsilon_{man}^2}{2}=\frac{1}{2}\cdot\frac{m_{man}\upsilon_{boy}^2}{4}\\ \frac{m_{man}(\upsilon_{man}+\Delta\upsilon)^2}{2}=\frac{m_{man}\upsilon_{boy}^2}{4}\end{cases}\Rightarrow\begin{cases}\upsilon_{man}^2=\frac{\upsilon_{boy}^2}{4}\\ (\upsilon_{man}+\Delta\upsilon)^2=\frac{\upsilon_{boy}^2}{2}\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\upsilon_{man}=\frac{\upsilon_{boy}}{2}\\ \upsilon_{man}+\Delta\upsilon=\frac{\upsilon_{boy}}{\sqrt{2}}\end{cases}\Rightarrow\begin{cases}\upsilon_{man}=\frac{\upsilon_{boy}}{2}\\ \upsilon_{boy}=\sqrt{2}\left(\upsilon_{man}+\Delta\upsilon\right)\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\upsilon_{man}=\frac{\sqrt{2}\left(\upsilon_{man}+\Delta\upsilon\right)}{2}\\ \upsilon_{boy}=\sqrt{2}\left(\upsilon_{man}+\Delta\upsilon\right)\end{cases}\Rightarrow\begin{cases}2\upsilon_{man}=\sqrt{2}\left(\upsilon_{man}+\Delta\upsilon\right)\\ \upsilon_{boy}=\sqrt{2}\left(\upsilon_{man}+\Delta\upsilon\right)\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}2\upsilon_{man}-\sqrt{2}\upsilon_{man}=\sqrt{2}\Delta\upsilon\\ \upsilon_{boy}=\sqrt{2}\left(\upsilon_{man}+\Delta\upsilon\right)\end{cases}\Rightarrow\begin{cases}\upsilon_{man}(2-\sqrt{2})=\sqrt{2}\Delta\upsilon\\ \upsilon_{boy}=\sqrt{2}\left(\upsilon_{man}+\Delta\upsilon\right)\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\upsilon_{man}=\frac{\sqrt{2}\Delta\upsilon}{2-\sqrt{2}}\\ \upsilon_{boy}=\sqrt{2}\left(\upsilon_{man}+\Delta\upsilon\right)\end{cases}\Rightarrow\begin{cases}\upsilon_{man}=\frac{\sqrt{2}\Delta\upsilon}{2-\sqrt{2}}\\ \upsilon_{boy}=\sqrt{2}\left(\frac{\sqrt{2}\Delta\upsilon}{2-\sqrt{2}}+\Delta\upsilon\right)\end{cases}\Rightarrow$$

$$\Rightarrow\begin{cases}\upsilon_{man}=\frac{\sqrt{2}\Delta\upsilon}{2-\sqrt{2}}\\ \upsilon_{boy}=\frac{2\sqrt{2}\Delta\upsilon}{2-\sqrt{2}}\end{cases}\Rightarrow\Rightarrow\begin{cases}\upsilon_{man}=2.414\ m/s\\ \upsilon_{boy}=4.828\ m/s\end{cases}$$

ANSWER:

$\upsilon_{man}=2.414\ m/s$

$\upsilon_{boy}=4.828\ m/s$