

QUESTION:

A ball P of mass 0.25kg loses one-third of its velocity when it makes a head-on collision with an identical ball Q at rest. After collision, Q moves off with a speed of 2m/s in the original direction of P. Calculate the initial velocity of P?

SOLUTION:

According to the momentum conservation law, the momentum before collision is equal to the momentum after the collision:

$$m_P v_P = m_B v_B + m_P v'_P$$

$$v'_P = \frac{2}{3} v_P - \text{the velocity of a ball P after the collision. Hence}$$

$$m_P v_P = m_B v_B + m_P \frac{2}{3} v_P$$

According to the law of energy conservation, kinetic energy before the collision is equal to the

$$\text{kinetic energy after the collision. } K_{\text{before}} = \frac{m_P v_P^2}{2}, \quad K_{\text{after}} = \frac{m_P \left(\frac{2}{3} v_P\right)^2}{2} + \frac{m_B v_B^2}{2}. \text{ Hence}$$

$$\frac{m_P v_P^2}{2} = \frac{m_P \left(\frac{2}{3} v_P\right)^2}{2} + \frac{m_B v_B^2}{2}$$

$$m_P v_P^2 = \frac{4}{9} m_P v_P^2 + m_B v_B^2$$

So,

$$\begin{cases} m_P v_P = m_B v_B + m_P \frac{2}{3} v_P \\ m_P v_P^2 = \frac{4}{9} m_P v_P^2 + m_B v_B^2 \end{cases} \Rightarrow \begin{cases} m_P \left(v_P - \frac{2}{3} v_P\right) = m_B v_B \\ m_P \left(v_P^2 - \frac{4}{9} v_P^2\right) = m_B v_B^2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{1}{3} m_P v_P = m_B v_B \\ \frac{5}{9} m_P v_P^2 = m_B v_B^2 \end{cases} \Rightarrow \begin{cases} \frac{1}{3} m_P v_P = m_B v_B \\ \frac{5}{9} m_P v_P^2 = m_B v_B^2 \end{cases} \Rightarrow \frac{\frac{5}{9} m_P v_P^2}{\frac{1}{3} m_P v_P} = \frac{m_B v_B^2}{m_B v_B} \Rightarrow$$

$$\Rightarrow \frac{5}{3} v_P = v_B$$

$$v_P = \frac{3}{5} v_B$$

$$v_P = 1.2 \text{ m/s}$$

ANSWER:

1.2 m/s