The horizontal distance between the balls doesn't change, because they move with the constant speed along X-axis.

So,

$$x = v_0 t_0$$

$$v_0 - initial\ velocity;\ t_0 - 1s.$$

Let's find vertical distance between the balls

$$y_1 = \frac{gt^2}{2}$$

$$y_1 - distance, which\ first\ ball\ had\ passed$$

$$y_0 = \frac{g(t - t_0)^2}{2}$$

$$y_0 - distance, which\ second\ ball\ had\ passed$$

So, distance between balls along Y-axis is:

$$y = y_1 - y_0 = \frac{gt^2}{2} - \frac{g(t - t_0)^2}{2} = \frac{gt_0(2t - t_0)}{2}$$

Using Pythagorean theorem:

$$d = \sqrt{y^2 + x^2} = \sqrt{\left(\frac{gt_0(2t - t_0)}{2}\right)^2 + (v_0 t_0)^2}$$

As, you can see $d(t)$ is increasing function, so

$$d_{min} = d(t_0) = \sqrt{\left(\frac{g{t_0}^2}{2}\right)^2 + (v_0 t_0)^2}$$

Where $v_0 - initial\ velocity;\ t_0 - 1s.$