A string of length 2l, obeying Hooke 's law, is stretched so that its extension is l. The speed of transverse waves in the string is v. If the string is further stretched so that the extension becomes 4l, what will be the speed of transverse waves in the string?

Solution

The phase velocity in the string is: $v = \sqrt{\frac{T}{\rho}}$,

where T is a tension in the string, ρ is the linear mass density of the string.

In the first case:

$$T = k\Delta l = kl, \rho = \frac{m}{2l+l} = \frac{m}{3l} \to v = \sqrt{\frac{3kl^2}{m}}.$$

In the second case:

$$T = k\Delta l = k4l, \rho = \frac{m}{2l+4l} = \frac{m}{6l} \to v` = \sqrt{\frac{24kl^2}{m}}.$$

So

$$\frac{v`}{v} = \frac{\sqrt{\frac{24kl^2}{m}}}{\sqrt{\frac{3kl^2}{m}}} = \sqrt{\frac{24}{3}} = \sqrt{8} \to v` = \sqrt{8}v = 2\sqrt{2}v.$$

Answer: $2\sqrt{2}v$.