Question 26214

1) First, let us use divergence theorem (Gauss-Ostrogradsky theorem) to find electric field outside the sphere as function of distance from center of sphere (Let r be that distance, and R be the radius of sphere):

$$\oint \vec{E}\,\vec{dS} = \frac{Q}{\varepsilon_0}$$

$$4\pi E r^2 = 4\pi\sigma\frac{R^2}{\varepsilon_0} \Rightarrow E(r) = \frac{r^2\sigma}{R^2\varepsilon_0} \quad .$$

From here obtain surface charge density($r = 12\,cm,\ R = 321\,cm$): $\sigma = \frac{E r^2 \varepsilon_0}{R^2} = 0.00031\frac{C}{m^2}$.

2) Let us find first formula for spherical capacitor with spheres of radius R and R_1 (R is fixed – radius of given sphere, R_1 is changeable):

By definition, $C = \frac{Q}{U}$, and $U = \oint \vec{E}\,\vec{dS} = \frac{Q}{4\pi\varepsilon_0}\int_R^{R_1}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{R_1}\right)$, so $C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{R} - \frac{1}{R_1}\right)}$.

Finally, taking $R_1 \to \infty$, obtain capacitance of sphere: $C = 4\pi\varepsilon_0 R = 1.33\cdot 10^{-11} F$.