

how can we derive the Schrödinger's wave equation?

Answer

The Schrödinger wave equation cannot be derived (at least not by any known means). Erwin Schrödinger literally guessed the equation (though his guess had reason behind it), and justification for its use can be attributed to the simple fact that it works.

Thought it was guess when Schrödinger formulated his wave equation he drew from a classical foundation. Total energy is equal to the kinetic energy plus the potential energy. Classically kinetic energy is expressed as $\frac{p^2}{2m}$, where p is momentum and m is the mass. The potential can be whatever you want it to be so we will just call it V, and then finally let's call the total energy E. We put this all together to get $\frac{p^2}{2m} + V = E$ but in terms of operators $p = [i * \frac{h}{2*\pi}] * \left(\frac{d^2}{dq_i^2}\right)$, where i is the square root of negative one, h is Planck's constant, π is 3.141529, and $\left(\frac{d^2}{dq_i^2}\right)$ is the second order differential with respect to space in generalized coordinates. So putting p back in our equation for energy we get $\left[-\frac{h^2}{4*\pi*m}\right] * \left(\frac{d^2}{dq_i^2}\right) + V = E$. Now comes the sort of complex part. In general the total energy of a system is defined by the Hamiltonian, let's call it H. Now, E is the eigenvalue of the Hamiltonian operator. Meaning that when H operates on some state it is measuring the total energy of that state. In quantum mechanics $H = \left[i * \frac{h}{2*\pi}\right] * \left(\frac{d}{dt}\right)$, where $\left(\frac{d}{dt}\right)$ is the first order differential with respect to time. Now putting this all together we get

$$\left[i * \frac{h}{2*\pi}\right] * \left(\frac{d}{dt}\right) = \left[-\frac{h^2}{4*\pi*m}\right] * \left(\frac{d^2}{dq_i^2}\right) + V.$$

That is Schrodinger's "picture" of the wave equation.