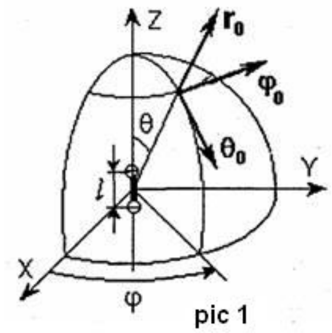
Condition:

Derive the expansion for the electric field at point on the axial line and equatorial line of electric dipole?



pic 1

Solution and answer:

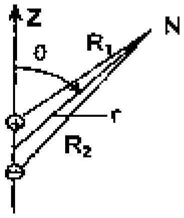
Electrostatic dipole is a system of two adjacent equal magnitude charges $+q$ and $-q$ (pic 1). Dipoles are characterized by a dipole moment

$$p = ql = l_0 ql, \qquad (1)$$

where l-vector pointing from the negative to the positive charge, the absolute value is equal to the distance between the charges l, and l_0 is the unit vector corresponding to the vector $l(l = l_o l)$.

If you bring together the charges, while increasing their value so that the vector p remained unchanged, we obtain a point or ideal dipole with the same momentum.

We calculate the electrostatic field of the dipole. We introduce a spherical coordinate system r, θ, φ, so that the polar axis passes through both charges and the origin is equidistant from them (pic 1). The potential created by the dipole, we find due to the principle of superposition as the sum of the potentials produced by the charges $+q$ and $-q$:



pic 2

$$u = \frac{q}{4\pi\varepsilon}\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \qquad (2)$$

where R_1 and R_2 - the distances from the charges $+q$ and $-q$ to the point at which the potential is calculated (pic 2):

$$R_1 = \sqrt{r^2 + \left(\frac{l}{2}\right)^2 - lr\cos\theta}, \quad R_2 = \sqrt{r^2 + \left(\frac{l}{2}\right)^2 + lr\cos\theta}.$$

In the calculation of the field, we assume that the distance r from the center of the dipole to the observation point is large compared to the distance between the charges l. Under this condition, we have the following approximate equality

$$r \mp \frac{l\cos\theta}{2} \cong \begin{cases} R_1 \\ R_2 \end{cases}, \ \frac{1}{R_1} - \frac{1}{R_2} \cong \frac{l\cos\theta}{2}. \qquad (3)$$

In this case (2) takes the form

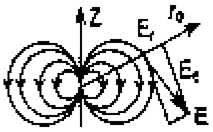
$$u = \frac{ql\cos\theta}{4\pi\varepsilon r^2} = \frac{(p, r_0)}{4\pi\varepsilon r^2}$$

where r_o - coordinate unit vector of variable r. To determine the electric field we use $E = -\nabla u$. The expression for the gradient in spherical coordinates is

$\nabla f = \frac{\partial f}{\partial r} r_0 + \frac{1}{r}\frac{\partial f}{\partial \theta}\theta_0 + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\varphi_0$. Using $\frac{du}{d\varphi} = 0$, we obtain

$$E = \frac{ql}{4\pi\varepsilon r^2}(r_0 2\cos\theta + \theta_0 \sin\theta).$$

Directions of the unit vectors r_0, θ_0 and φ_0 are shown in pic 1. As can be seen, the vector of the electric field created by an electrostatic dipole, is independent of the angle φ (field has axial symmetry) and has two components:



pic 3

$$E_r = \frac{ql\cos\theta}{2\pi\varepsilon r^3}, \quad E_0 = \frac{ql\sin\theta}{4\pi\varepsilon r^3}.$$

The field lines are shown in pic 3.

<u>For the electric field at the point on the axial line ($\theta = 0$):</u>

$$E_r = \frac{ql}{2\pi\varepsilon r^3}, \quad E_0 = 0.$$

<u>For the electric field at the point on equatorial line ($\theta = \frac{\pi}{2}$):</u>

$$E_r = 0, \quad E_0 = \frac{ql}{4\pi\varepsilon r^3}.$$