b) Suppose a spring-mass system has 18Nm 1 − k = and m = 0.71kg. The system is oscillating with an amplitude of 54 mm. (i) Determine the angular frequency of oscillation. (ii) Obtain an expression for the velocity v of the block as a function of displacement, x and calculate v at x = 34 mm. (iii) Obtain an expression for the mass's distance | x | from the equilibrium position as a function of the velocity v and calculate | x | when v = 0.18 ms−1. (iv) Calculate the energy of the spring-mass system.

Solution and answer

(i) The frequency of oscillation is:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{18}{0.71}} = 5.035\ s^{-1}$$

(ii) According to the conservation of energy law the energy of the spring-mass system consist of the kinetic energy of the block and the potential energy of the spring

$$E_{sms} = \frac{mv^2}{2} + \frac{kx^2}{2}$$

At 54 mm spring compression (amplitude), the energy is all Potential Energy (PE) in the spring:

$$PE = \frac{kx_{amp}^2}{2} = 0.5 * 18 * 0.054^2 = 0.026\,J = 26\ mJ$$

The velocity v of the block as a function of displacement

$$v = \sqrt{\frac{2E_{sms} - kx^2}{m}}$$

$$v(34\text{ mm}) = \sqrt{\frac{2 * 0.026 - 18 * 0.034^2}{0.71}} = 0.2\frac{m}{s}$$

(iii) An expression for the mass's distance | x | from the equilibrium position as a function of the velocity v

$$|x| = \sqrt{\frac{2E_{sms} - mv^2}{k}}$$

$$x\left(0.18\frac{m}{s}\right) = \sqrt{\frac{2 * 0.026 - 0.71 * 0.18^2}{18}} = 0.040\ m = 40\ mm$$

(iv) The energy of the spring-mass system is Potential Energy (PE) in the spring at amplitude

$$E_{sms} = PE = \frac{kx_{amp}^2}{2} = 0.5 * 18 * 0.054^2 = 0.026\,J = 26\ mJ$$