QUESTION:

A truck is traveling at 11.1 m/s down a hill when the brakes on all four wheels lock. The hill makes an angle of 15.00 with respect to the horizontal. The coefficient of kinetic friction between the tires and the road is 0.750. How far does the truck skid before coming to a stop?

ANSWER

According to the work-energy theorem:

$$A = 0 - (E_{kin} + E_{pot})$$

$$E_{kin} = \frac{m_{truck}\upsilon_{truck}^2}{2}$$

$$E_{pot} = m_{truck}gh$$

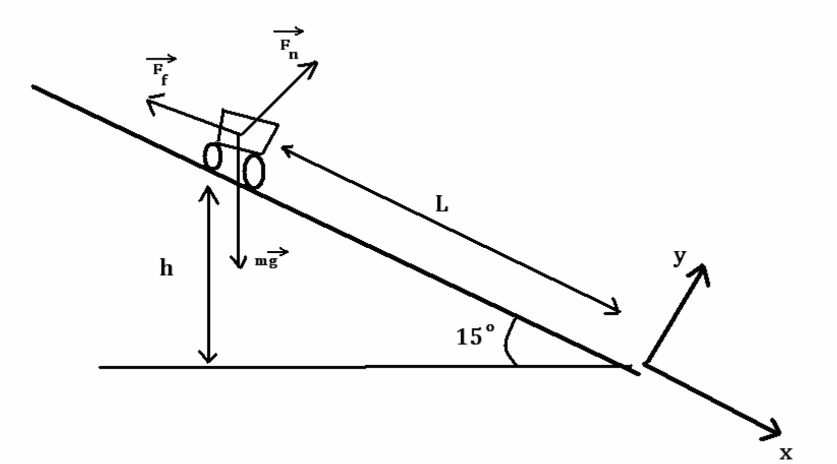
$$A = F_f \cdot L\cos(180^o) = -F_f L$$

$$h = L\sin 15^o$$

Hence

$$F_f L = \frac{m_{truck}\upsilon_{truck}^2}{2} + m_{truck}g \cdot L\sin 15^o$$

Let's sketch a free-body diagram for the truck:



Since $F_f = \mu F_n$, and

$$F_n - m_{truck}g\cos 15^0 = 0 \text{ (projection on y-axis)}$$

$$F_n = m_{truck}g\cos 15^0$$

Hence, friction force is

$F_f = \mu F_n = \mu m_{truck}g\cos 15^0$, and

$$F_f L = \frac{m_{truck}\upsilon_{truck}^2}{2} + m_{truck}g \cdot L\sin 15^o$$

$$\mu m_{truck}g\cos 15^0 \cdot L = \frac{m_{truck}\upsilon_{truck}^2}{2} + m_{truck}g \cdot L\sin 15^o$$

$$\mu g\cos 15^0 \cdot L = \frac{\upsilon_{truck}^2}{2} + g \cdot L\sin 15^o$$

$$L \cdot g \cdot \cos 15^0 \cdot (\mu - tg 15^0) = \frac{\upsilon_{truck}^2}{2}$$

$$L = \frac{\upsilon_{truck}^2}{2 \cdot g \cdot \cos 15^0 \cdot (\mu - tg 15^0)}$$

$$L = \frac{11.1^2}{2 \cdot 9.8 \cdot \cos 15^0 (0.75 - tg 15^0)}$$

$$L = 13.5 \ \ m$$