due to the sun-earth distance the average temperature of earth is maintained at 255k. Derive this temperature of earth and state any assumptions made and state the variables used.

Solution

Using the Stefan-Boltzmann law and energy conservation law for Earth's radiation balance

$$\alpha\pi R_{earth}^2 L = 4\pi\varepsilon R_{earth}^2 \sigma T^4$$

Where $1-\alpha$ is reflection coefficient, ε is emissivity, T is temperature of earth, R_{earth} is radius of earth, σ is Stefan-Boltzmann constant, L is solar constant.

$$\alpha L = 4\varepsilon\sigma T^4$$

$$T = \sqrt[4]{\frac{\alpha L}{4\varepsilon\sigma}}$$

I defined constant as

$$\alpha \approx 0.6$$

$$\varepsilon \approx 0.9$$

$$L = 1360 W/m^2$$

Answer

$$T = \sqrt[4]{\frac{0.7 * 1360 W/m^2}{4 ** 5.67 * 10^{-8}}} K \approx 252 K$$