An object is thrown vertically upwards from the surface of the earth, with a speed(VO). If its speed at a height (h) is(V),then show that

V0square - Vsquare=2GM/Rsquare(h/1+h/r) =2gh/(1+h/r)

Solution

Using the energy conservation law (R_{earth} is the radius of Earth, M is the mass of Earth) we obtain

$$\frac{mv_0^2}{2}-\frac{mv^2}{2}=\frac{GmM}{R_{earth}}-\frac{GmM}{R_{earth}+h}\Rightarrow$$

$$v_0^2-v^2=\frac{2GMh}{R_{earth}(R_{earth}+h)}$$

We have

$$\frac{GM}{R^2{}_{earth}}=g\Rightarrow$$

$$v_0^2-v^2=\frac{2g}{(1+\frac{h}{R_{earth}})}$$

Answer

$$v_0^2-v^2=\frac{2g}{(1+\frac{h}{R_{earth}})}$$