

You and a friend are going to a river, where they have a rope swing. Your friend, who has a mass of 60 kg, hops on the end of the rope. You tug horizontally on your friend with a force of 300 N, pulling them back until you can't move them any more. Then you let go and watch them swing out over the river. When the rope swing is at its farthest point your friend lets go, eventually splashing into the river. If the rope is 30 m long, what total distance in m did your friend travel from her starting point to where she let go?

Solution

Since there is no air resistance, the final position of my friend is the same as the initial position of my friend. As the rope swings from its position to its final position, my friend moves an arc of a circle. The total distance which my friend moves is the length of the arc. To determine the length of the arc, we need to determine the angle of the arc, and the circumference of the circle.

$$\text{Circumference} = 2 * \pi * 30 = 60 * \pi \approx 188.5 \text{ meters}$$

$$\text{Total angle of circle} = 360^\circ$$

$$\text{Length of arc} / 60 * \pi = \frac{\theta}{360}.$$

We need to determine the total angle that she moved.

My friend has 3 forces which affect his acceleration, the tension in the rope, her weight, and the 300 N force. The vertical component of the tension in the rope supports the weight of my friend. The horizontal component of the tension in the rope is equal to the 300 N force.

Vertical component of the tension = $T * \cos \theta$, θ is the angle between the rope and vertical.

$$\text{Weight of friend} 60 * 9.8 = 588 \text{ N}$$

$$\text{Eq#1: } T * \cos \theta = 588$$

$$\text{Horizontal component of the tension} = T * \sin \theta$$

$$\text{Eq#2: } T * \sin \theta = 300$$

Divide Eq#2 by Eq#1

$$\tan \theta = 300 \div 588$$

$$\theta = 27.03^\circ$$

This is the angle between the rope and vertical. As the rope swings 27.03° , my friend moves from her highest point to her lowest point. As the rope swings a second 27.03° , my friend moves from her lowest point to her highest point. The total angle my friend has moved is 54.06°

$$\text{Length of arc} / 60 * \pi = \frac{\theta}{360}$$

$$\text{Length of arc} / 60 * \pi = \frac{54.06}{360}$$

$$\text{Length of arc} = 60 * \pi * \frac{54.06}{360} = 28.3 \text{ m}$$

This is the total distance that she traveled.

Answer: 28.3 m.