QUESTION:

Suppose a spring-mass system has 18Nm k = and m = 0.71kg. The system is oscillating with an amplitude of 54 mm. (i) Determine the angular frequency of oscillation.

(ii) Obtain an expression for the velocity v of the block as a function of displacement, x and calculate v at x = 34 mm. (iii) Obtain an expression for the mass's distance | x | from the equilibrium position as a function of the velocity v and calculate | x |when v = 0.18 ms−1. (iv) Calculate the energy of the spring-mass system.

SOLUTION:

The angular frequency of oscillation $\omega = \sqrt{\frac{k}{m}} = 5.05\ \frac{rad}{s}$

The velocity of the block is $\upsilon(t) = \frac{dx}{dt} = \frac{d}{dt}(x_0 \sin(\omega t + \phi_0)) = \omega x_0 \cos(\omega t + \phi_0)$

Since $\cos^2\alpha + \sin^2\alpha = 1,\ x_0\cos(\omega t + \phi_0) = x_0\sqrt{1 - \sin^2(\omega t + \phi_0)} = \sqrt{x_0^2 - x(t)^2}$, therefore

$\upsilon(t) = \omega\sqrt{x_0^2 - x(t)^2}$. x_0= 54 mm – amplitude of oscillation

When x = 34 mm $\upsilon = \omega\sqrt{x_0^2 - x^2} = 0.2119\ m/s$

$$x(t) = x_0\sin(\omega t + \phi_0) = x_0\sqrt{1 - \cos^2(\omega t + \phi_0)} = x_0\sqrt{1 - \left(\frac{\upsilon(t)}{\omega \cdot x_0}\right)^2}$$

When $\upsilon = 0.18\ m/s$

$x(t) = 40.56\ mm$

The energy of the system is $E = \frac{k \cdot x_0^2}{2} = 0.0262\ J$